Aeration and Deaeration Processes in Large Power Plant Condensers

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Absorption of noncondensible gases by the condensate in large power plant condensers is extremely deleterious to plant components. Unfortunately, the absorption process is one of the most complicated and insufficiently researched problems of contemporary hydrodynamics. Additionally, removal of noncondensible gases dissolved in condensate at very low concentrations is extremely difficult from the technical as well as the theoretical points of view.

This paper attempts to address the above problems by establishing physical dependencies governing the aeration and deaeration processes during condensation, developing a means of estimating deaerating efficiency of a condensing apparatus, and formulating directions for further theoretical and experimental research in the area of condenser deaeration. The authors also indicate a future research path to determine optimal situation to achieve maximum condensation with minimum oxygen absorption.

1. PURPOSE, ASSUMPTIONS, AND LIMITATIONS

The purpose of this paper is to establish physical dependencies governing the aeration and deaeration processes in main condensers.

Oxygen concentrations approaching even 10 ppb play a significant role in corrosion processes leading, for example, to the long-term degradation of pressurized water reactor (PWR) steam generators. Therefore, it appears to be essential to try to determine the stages of the condensation process and various factors associated with these stages that contribute to condensate aeration. However, such an attempt will necessarily fail if it is applied to the complex process of the air/steam/condensate interactions within the huge volume and complicated geometry of a contemporary condenser. The complexity of the hydrodynamic situation inside an actual condenser necessitates simplifications and modeling. For the purpose of this study we therefore employ the model depicted in Fig. 1. This model is sufficiently close to the actual situation and simple enough to allow for theoretical analyses.

In accordance with this model, the airless steam flow is supplied to a closely spaced tubing system characterized by the distance h between adjacent tubes. This distance is equal to or less than the individual tube radius R. As a result of the condensation on the first few rows of tubes, a condensate film with thickness $d(x)$ is formed. In general, the film thickness

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*Deceased.

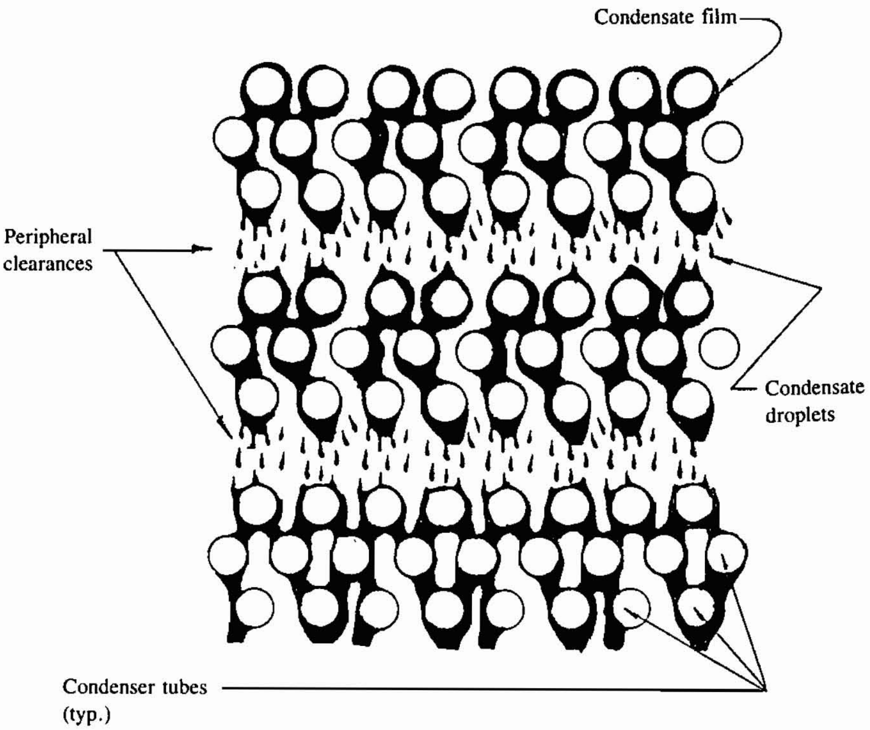

Figure 1 Tube bundle model.

increases as the film flows down and more steam condenses on it. It will be shown in Secs. 3 and 5 that the distance between adjacent tubes is small enough to allow the film to move downward without losing its continuity. However, it is necessary to note that in reality film trajectories may be rather complex. Moreover, film rupture is quite possible and most probably takes place on peripheral tubes as a result of the impact by high-velocity steam flows. The latter subject will be addressed in Sec. 6. Additionally, condenser tube-bundle configurations contain a number of peripheral clearances (Fig. 1). Each such clearance represents a break point where the film continuity may be interrupted. Therefore, for the purposes of this study, the vertical length of a tube bundle is divided into N continuous film descents of identical length L.

Depending on the actual distance between the lowest row of tubes and the average hot well condensate level, two entirely different possibilities for film behavior (Fig. 2) could be postulated:

1(*a*). The film is being broken into rivulets, then into droplets, and then the condensate rain reaches the hot well; or

2(*b*). The condensate reaches the hot well as a system of curtains carrying the steam along.

It is practically impossible to decide which case is more realistic without an experiment. However, it is quite reasonable to assume that the condensate aeration process is almost insensitive to the type of film behavior. This assumption will be verified later (see Sec. 5).

The film is interacting with the steam flow moving with velocity U over the entire tube-bundle height. This velocity is normally within the range of 120–250 fps. It is therefore assumed that air absorption by films, rivulets, and droplets takes place over the entire length of the condensate descent.

This completes the description of the hydrodynamic processes within the assumed simplified model. This model appears to be adequate for estimation of the maximum air quantity captured by condensate throughout the entire condensation process. The model allows for determination of the relative importance of various condensation stages to the maximum condensate air content and for formulation of corrective measures to reduce the condensate air concentration.

It is necessary to note here that the overall evaluation of such a complex process should never be limited to theoretical analyses only. Therefore, some basic measurements required to verify the adequacy of the theoretical approach will be specified and recommended in this paper.

Finally, it is absolutely essential to define limitations of the suggested model. These limitations are as follows:

1. The model is not expected to produce results more accurate than order-of-magnitude values.
2. The models does not account for three-dimensional nonuniformity and complex geometry of a condenser.
3. The model assumes no oxygen in the low-pressure turbine exhaust steam flow.

2. GAS SOLUBILITY IN LIQUID

An isolated liquid/soluble gas system reaches a state of statistical or thermodynamic equilibrium after a certain period of time (relaxation time). In this state, the number of gas molecules crossing the phase interface per unit of time (specific speed of solubility j_g) is equal to the number of liquid molecules crossing the phase interface in the opposite direction, j_{sol}:

$$j_g \rightleftharpoons j_{sol} \tag{1}$$

It is also obvious that

$$j_g \approx n_g \approx p_g$$

and

$$j_{sol} \approx C_{sol}^{eq}$$

where n_g and n_{sol}^{eq} are the numbers of molecules in the gas and liquid phases per unit volume under equilibrium conditions, respectively, and p_g is the partial gas pressure. Therefore, Eq. (1) represents Henry's law, which can be written as

$$\gamma C_{sol}^{eq} = p_g \tag{2}$$

Henry's law, however, is not applicable to dissolving processes under conditions far from equilibrium, particularly when the time of contact is substantially smaller than the required relaxation time *T*. In the latter situation the gas concentration in the liquid does not reach its equilibrium value. Therefore, in order to be able to deal effectively with such situations, it is necessary to invoke the kinetic laws of solubility.

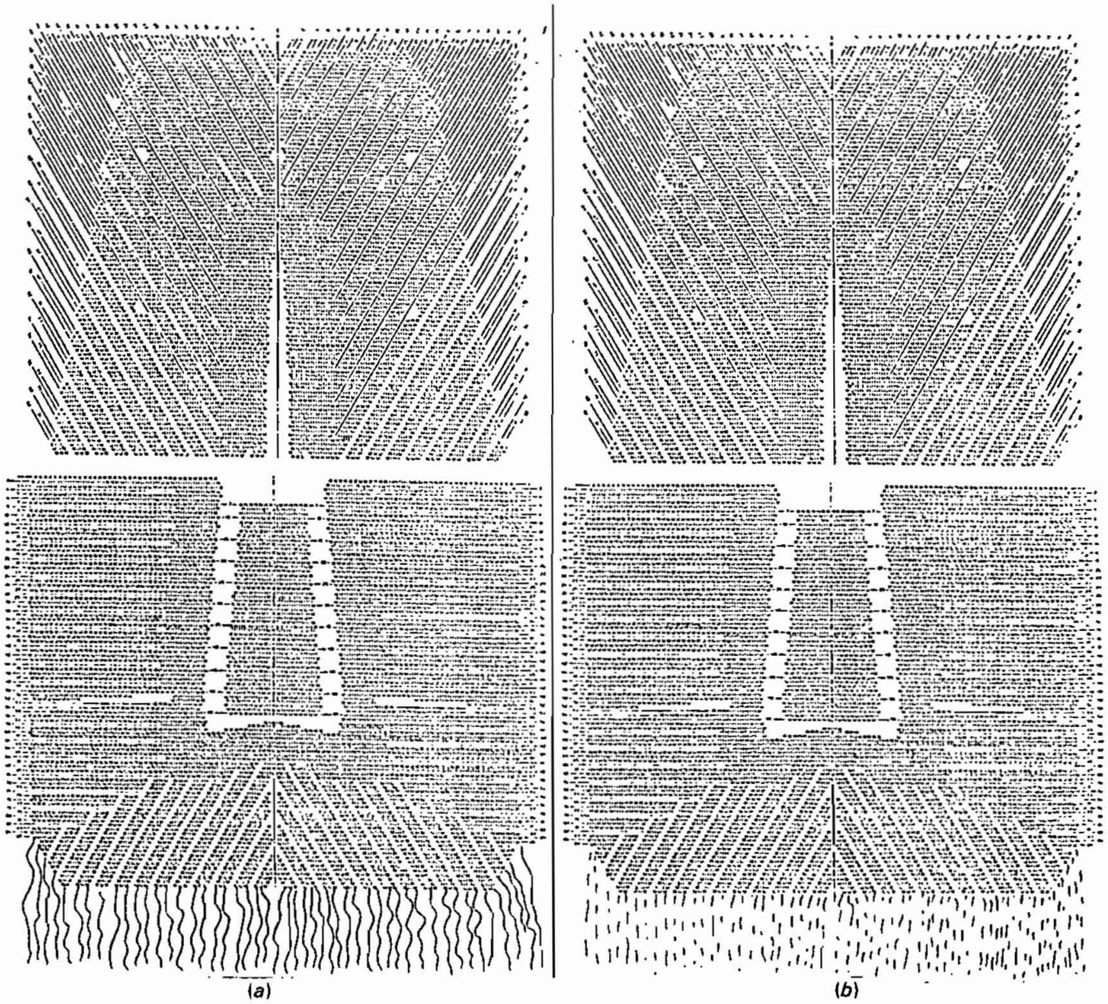
(a) (b)

Figure 2 Condensate film behavior models.

Generally, a dissolving process consists of the three following stages:

1. Transfer of gas particles from a bulk to a gas/liquid interface.
2. Gas particle crossing of a gas/liquid interface, or actual gas particle penetration into liquid.
3. Removal of dissolved gas particles from a phase interface into the volume of liquid.

Each of the above stages proceeds at its own speed, as shown below. The overall speed of the dissolving process is therefore limited by the speed of the slowest stage.

When easily soluble gases are involved, the effective surface resistance of the liquid is minimal. In other words, the overall speed of the dissolving process is relatively high. This is exactly the case for air dissolving in water. Therefore, there are only two stages that may limit the overall speed of the process:

1. Stage 1, when the air particle supply to the interface is the limiting factor; and
2. Stage 3, when the removal of the dissolved gas from the interface into the volume of liquid limits the overall speed of the dissolving process.

In stage 1, every air particle approaching the phase interface is absorbed without delay. Therefore, the concentration of the dissolving component C will eventually be established at the liquid surface S:

$$C|_S = 0 \tag{3}$$

In stage 3, a surplus of air particles is transferred to the surface S. As a result, the equilibrium condition will eventually be established at this surface:

$$C|_S = \gamma C_{\text{gas}}^{\infty} \tag{4}$$

where C_{gas}^{∞} = air concentration far enough from the interface. The difference between Eq. (4) and Henry's law [Eq. (2)] is rather significant. From Henry's law the concentration is at equilibrium over the entire liquid volume, whereas in Eq. (4) this equilibrium is established only at the liquid surface.

It should also be noted here that in the case of very low process pressure and easily soluble gases, the coefficient γ is of the order of unity.

3. CONDENSATE FILM

The assumption formulated in Sec. 1 regarding the formation of condensate film flowing down from the tube bundle should be analyzed carefully. The fact that film condensation (as opposed to dropwise condensation) takes place in actual situations is rather obvious. However, the further evolution of that film is unclear. Four scenarios are possible here:

1. The film moves downward successively from one tube to another, remaining continuous as long as it remains within the closely spaced tubing system. The model formulated in Sec. 1 and depicted in Fig. 1 is based on this assumption.
2. The film is atomized by the gas (steam) flow.
3. The film loses continuity upon separation from each tube.
4. The film is totally unstable and breaks into droplets on the surface of each individual tube.

We shall discuss each of the above cases.

It is well known that film flow down a vertical or inclined solid surface is unstable and has a tendency to break into droplets or rolls [1]. This film instability is most clearly demonstrated on a convex surface. However, the latter effect will manifest only when the film thickness is of the same order of magnitude as the convex curve radius. Additionally, the length of the film should reach at least 50–100 film thicknesses. The condensate film on any individual tube falls far short of meeting these conditions. Therefore, scenario 4 can be ruled out as totally unrealistic.

The process of velocity profile reconstruction commences immediately in the liquid curtain when the latter leaves the tube surface. As a result, the curtain moving in the intertube space is even less stable than the film on the solid surface. However, the velocity profile reconstruction progresses rather slowly. Therefore, the Poiseuillian velocity profile transforms into an almost rectilinear one, typical for the curtain, over a length of approximately 30–50 thicknesses. This length substantially exceeds the normal distance between adjacent condenser tubes. Therefore, the liquid curtain in the condenser intertube space does not have enough time to appear, let alone be destroyed. The latter consideration rules out scenario 2.

A serious threat to the film integrity is posed by the possibility of film atomization by a steam flow moving at a high velocity. However, the probability of film atomization is minimal due to the very low steam density that exists at the very low condenser operating pressures. This case will be analyzed in Sec. 5.

Let us consider now the behavior of the condensate film appearing on the solid surface. The initial film thickness h_0 is small compared with the tube radius R. Therefore, it is a good approximation to consider each film surface element as flat. The film flows down under the force of gravity. If the velocity is U, the corresponding Reynolds number is

$$\mathrm{Re}_0^{\ell} = \frac{Uh_0}{\nu_\ell}$$

where Re_0^{ℓ} = condensate Reynolds number
ν_ℓ = condensate kinematic viscosity at temperature approaching $T = (T_{\text{tube}} + T_{\text{gas}})/2$
T_{tube} = tube surface temperature
T_{gas} = film surface temperature

The steady flow of the thin film establishes itself rapidly, over a length equal to only a few film thicknesses. However, film thickness continues to increase as a result of the ongoing massive condensation process and the influence of steam flow. The film thickness increase associated with the heat balance of the moving film is unquestionably an extremely interesting subject for a special investigation; however, it is outside the scope of this study. Influence of steam flow on the film is discussed in Sec. 6.

If $h(x)$ is the film thickness for the film that has covered a distance x from the point of its creation, the local Reynolds number $\mathrm{Re}_x = U(x)h(x)/\nu_\ell$ and increases with the distance. When Re ≈ 20, waves start to appear on the film surface. Further development of these waves leads to the loss of film stability and to its breakdown into the system of rolls. Complete film breakdown takes place before the Reynolds number Re_x reaches the critical values of approximately 1600–1800, after which the film flow becomes turbulent. Based on the above, it appears that film rupture may occur not as a result of hydraulic instability, but due only to the presence of large intertube ligaments that might be a part of the condenser geometry. It is therefore possible to assume (as a first approximation) that the film flow is laminar and that its thickness is fixed. An effect that the wavy character of the flow and its variable thickness makes on the mass transfer will be addressed later (see Sec. 5).

It is reasonable to assume that the convective diffusion of gas is the slowest and, therefore, the limiting aspect of film flow aeration. There are two reasons supporting this assumption. First, the steam velocity with respect to the film substantially exceeds the velocity of the film descent. Second, steam impurities (air in our case) are drawn along with the steam during the massive condensation process. The latter further accelerates the transport of impurities into the liquid phase.

The velocity distribution in the laminar film flowing down under the force of gravity along the solid surface is easily derived from the Navier-Stokes equation with appropriate boundary conditions imposed and can be written as:

$$v_x = \frac{g'}{2\nu_\ell}(2h_0 y - y^2) \tag{5}$$

$$v_y = 0$$

The film surface is assumed flat with $y = h_0$, and the solid surface is chosen as a plane $y = 0$. Here $g' = g \cos\theta$, where g is the acceleration due to gravity and θ is the angle of inclination of the solid surface. The average velocity across the film section can be expressed as:

$$\bar{U} = \frac{1}{h_0}\int_0^{h_0} v_x\,dy = \frac{g' h_0^2}{3\nu_\ell} \tag{6}$$

Equations (5) and (6) will be used in Sec. 5.

4. CONVECTIVE DIFFUSION IN MOVING FLUIDS

In accordance with the general laws of the theory of convective diffusion in fluids, if $C(x, y, z, t)$ is the concentration of a dissolved substance changing from point to point in space and time, the flux of substance is inevitable. If the concentration C is low enough, the flux density in the moving media can be presented as

$$\mathbf{j} = C\mathbf{V} - D\nabla C \tag{7}$$

or, in components,

$$\mathrm{j}_i = CV_i - D\frac{\partial C}{\partial x_i} \qquad (i = x, y, z) \tag{8}$$

Here the vector $\mathbf{j}$ is the mass flux density, i.e., the number of particles crossing an imaginary fluid plane with an area of 1 cm^2 per 1 s; $\mathbf{v}$ is the velocity of liquid; and D is the coefficient of molecular diffusion.

The first term in Eq. (7) represents the convective (passive) transfer of substance by the moving fluid. The second term represents the molecular flow directed from places with high to places with low concentrations.

Let us consider some arbitrary volume V in a body of fluid and find the net flow of particles of the dissolved substance entering and leaving this volume per unit time. The change in the number of particles in the volume V is equal to $Q = \partial N/\partial t$. The number of particles passing through the surface S surrounding volume V per 1 s is $\int \mathbf{j}\, d\mathbf{S}$, where the integral is taken over the entire surface, and the direction outward from the surface is chosen as the positive direction for the normal vector.

Equating the integral flux Q to the change in the number of particles in the volume V, we obtain:

$$\int \frac{\partial C}{\partial t}\, dV = -\int \mathbf{j}\, d\mathbf{S}$$

The integral on the right side of the above equation can be converted according to the Gauss theorem so that

$$\int \left(\frac{\partial C}{\partial t} + \operatorname{div} \mathbf{j} \right) dV = 0 \tag{9}$$

In view of the arbitrariness of volume V, the above condition is satisfied only if

$$\frac{\partial C}{\partial t} + \operatorname{div} \mathbf{j} = 0 \tag{10}$$

or, substituting Eq. (7),

$$\frac{\partial C}{\partial t} + \operatorname{div}(C\mathbf{V}) - D \operatorname{div} \operatorname{grad} C = 0 \tag{11}$$

Remembering that in the incompressible liquid div $\mathbf{V} = 0$ and neglecting the dependency of D on concentration (and, therefore, on coordinates), we obtain:

$$\frac{\partial C}{\partial t} + (\mathbf{V} \operatorname{grad})\, C = \operatorname{div} D \operatorname{grad} C \tag{12}$$

Equation (12) describes the mass transfer process in a moving liquid and is called "the equation of convective diffusion." Fluid velocity $\mathbf{V}$ in Eq. (12) is assumed to be a known function of coordinates and time. Since the fluids under consideration are characterized by very low concentrations of various impurities, the reverse influence of concentration change on velocity distribution can be safely neglected. Moreover, since the case under consideration is steady-state, or almost steady-state, the dependence of velocity and concentration on time can also be neglected. As a result, Eq. (12) is further simplified to

$$(\mathbf{V} \operatorname{grad})\, C = D\nabla C \tag{13}$$

or, in coordinates,

$$V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y} + V_z \frac{\partial C}{\partial z} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \tag{14}$$

To find a full solution of the problem, i.e., to find a field of concentration $C(x, y, z)$, we must also know the proper boundary and initial conditions.

In order to clarify the qualitative picture of the mass transfer process, it is necessary to transform Eq. (14) to the dimensionless form. If L is a characteristic dimension of a system (a tube radius, a size of the dissolving body, etc.) and U represents a characteristic fluid velocity, then by introducing dimensionless ratios

$$X_i = \frac{x_i}{L} \qquad V_i = \frac{v_i}{U} \qquad C = \frac{c}{C^\infty}$$

it is easy to rewrite Eq. (8) as follows:

$$\left(V_x \frac{\partial C}{\partial X} + V_y \frac{\partial C}{\partial Y} + V_z \frac{\partial C}{\partial Z} \right) - \frac{1}{\mathrm{Pe}} \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} + \frac{\partial^2 C}{\partial Z^2} \right) = 0 \tag{15}$$

where $\mathrm{Pe} = UL/D$ is the dimensionless Peclet number. It is easy to see that

$$\mathrm{Pe} = \frac{UL}{D} = \frac{UL}{\nu} \frac{V}{D} = \mathrm{Re} \left(\frac{\nu}{D} \right) = \mathrm{Re} \cdot \mathrm{Pr} \tag{16}$$

where the diffusion Prandtl number (or Schmidt number) is

$$\mathrm{Pr} = \frac{\nu}{D} \tag{17}$$

The first term in parentheses in Eq. (15) characterizes the convective mass transfer into the moving fluid. The second term in parentheses in Eq. (15) characterizes the mass transfer by means of molecular diffusion. All dimensionless terms in Eq. (15) have, generally speaking, an order of magnitude of unity. Therefore, the relationship between the diffusion and convective mass transfers is characterized only by the dimensionless number in Eq. (15), i.e., the Peclet number. In the case under consideration, the Peclet number is always substantially greater than 1. In the case of the steam phase, $\mathrm{Pr} \simeq 1$, but $\mathrm{Re} \gg 1$ and, therefore, $\mathrm{Pe} \gg 1$. For a liquid film Re can vary approximately within the range 1–10–500, but $\mathrm{Pr} \simeq 10^3$. Therefore, $\mathrm{Pe} \gg 1$. This means that in our particular case the diffusion mass transfer part of Eq. (15) can be neglected, and the entire equation can be simplified to

$$V_x \frac{\partial C}{\partial X} + V_y \frac{\partial C}{\partial Y} + V_z \frac{\partial C}{\partial Z} = 0 \tag{18}$$

It is obvious that C = const is a solution of Eq. (18). However, it is also obvious that the solution C = const does not satisfy the boundary conditions specified in Sec. 2 and therefore is not an acceptable solution of the mass transfer equation. Actually, until the mass transfer process is complete (which in our case means that the equilibrium air concentration is established throughout the entire phase volume), concentrations at the phase interfaces and in the bulk of the phase will differ. This seeming contradiction could be easily removed by taking into account the fact that, in the actual process, the concentration in the immediate vicinity of the interface changes very rapidly from point to point. This means that here the derivatives of concentration with respect to distance are very large. In other words, mass transfer is determined by molecular diffusion as well as by convection near the interface. As a result, the terms in the second set of parentheses in Eq. (15) are so large near the special surfaces (such as the phase interface) that, despite the small coefficient 1/Re associated with the parentheses, the second term in parentheses is still of the same order of magnitude as the first term in parentheses. This means that if $\mathrm{Pe} \gg 1$, the concentration is practically constant throughout the entire fluid volume except in the phase interface area. The latter is characterized by the presence of a very thin layer, called the diffusion boundary, where practically all concentration changes take place. In this region mass transfer takes place by means of both molecular diffusion and convection.

The diffusion boundary layer plays a role similar to that of Prandtl's hydrodynamic boundary layer. Moreover, the thickness of the diffusion boundary layer δ' is correlated with the thickness of Prandtl's hydrodynamic boundary layer δ_0 by the equations

$$\delta' \simeq \left(\frac{\nu}{D}\right)^{1/3} \delta_0 \qquad \text{for liquids} \tag{19}$$

$$\delta' \simeq \delta_0 \qquad \text{for gases} \tag{20}$$

When the fluid is moving along the solid surface, the thickness of the hydrodynamic boundary layer increases proportionally to the distance x:

$$\delta_0 \simeq \sqrt{\frac{x}{U\nu}} \tag{21}$$

This particular fact has necessitated additional consideration of the mass transfer process in thin moving films. Generally speaking, usual expressions derived for deep fluids are no longer applicable to cases in which the boundary-layer thickness becomes comparable to the film thickness. In cases of sufficiently long films (i.e., with large values of x), the latter situation appears to be quite realistic.

5. CONVECTIVE DIFFUSION IN THIN LIQUID FILMS

In accordance with the previous assumptions regarding laminar and waveless liquid film flow, along a solid surface of small convexity, the convective diffusion equation is derived by combining Eqs. (5) and (14) as follows:

$$\frac{g'}{2\nu_\ell}(2h_0 y - y^2)\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{22}$$

In the case where the absorption occurs into an initially pure liquid and eqilibrium concentration C_{eq} is reached fast enough on the liquid surface, the system of boundary conditions can be written as:

$$C(0, y) = 0 \quad C(x, 0) = 0 \quad C(x, h_0) = C^{\mathrm{eq}} \tag{23}$$

It is our aim to determine the diffusion flux density (i.e., the number of particles crossing 1 cm^2 of surface per 1 s):

$$j = D\left(\frac{\partial C}{\partial x}\right)\bigg|_{y=h_0} \tag{24}$$

If $j(x)$ is known, the total flux can be easily determined as follows:

$$J = \int j(x)\, dS \tag{25}$$

It is now convenient to introduce dimensionless variables:

$$\xi = \frac{x}{L} \qquad \eta = \left(1 - \frac{y}{h_0}\right)\sqrt{\text{Pe}} \tag{26}$$

In our particular case, by definition, the Peclet number is

$$\text{Pe} = \frac{\bar{U}h_0}{\nu_t} \approx \frac{g' h_0^3}{\nu_t D} \tag{27}$$

Using the variables ξ and η, Eq. (22) and boundary conditions (23) can be written as:

$$\alpha(1 - \text{Pe}\cdot\eta^2)\frac{\partial C}{\partial \xi} - \frac{\partial^2 C}{\partial \eta^2} = \beta\frac{\partial^2 C}{\partial \xi^2} \tag{28}$$

$$C(0, \eta) = 0$$

$$C(\xi, \sqrt{\text{Pe}}) = 0 \tag{29}$$

$$C(\xi, 0) = C_{\text{eq}}$$

where

$$\alpha = \frac{h_0}{L} \quad \text{and} \quad \beta = \frac{1}{\text{Pe}}\left(\frac{h_0}{L}\right)^2 \tag{30}$$

It is obvious that α and β are very small compared to unity and that $\beta \ll \alpha$.

It is necessary to note here that in the case of a substantially deep fluid it is possible not to account for the velocity distribution in the depth and to restrict ourselves to the approximation of the free surface velocity. This leads to an essential simplification of the problem. In the case of a thin film, however, it is not at all obvious. Substitution of a real velocity profile by a linear profile could result in a substantial qualitative error for a thin-film situation. Therefore, it appears to be necessary to carry out consistent calculations in order to find the solution to the set of equations (28)–(29). The solution of this problem can be found by expanding into an infinite series in powers of the small parameter β:

$$C(\xi, \kappa\eta, \alpha) = C^{(0)} + \beta C^{(1)} + \beta^2 C^{(2)} + \dots \tag{31}$$

Here the functions $C^{(0)}$, $C^{(1)}$, . . . , etc., are dependent not only on the coordinates but also on the parameter α.

The goal here is to determine densities of the flux of particles moving from the surface into the film. The calculations show, however, that an attempt to find the terms of the series (31) by means of the usual perturbative method (the method of sequential approximations) leads to a mathematically incorrect expression for the diffusion flux. The expression for the normal derivative of concentration in the first approximation is proportional to $1/\xi^{1/2}$. This means that the flux along the entire surface [expressed by the integral (25)] diverges (tends to infinity). Obviously, this does not make any physical sense. This paradox is well known from similar problems, in particular from Prandtl's theory of the hydrodynamic boundary layer.

Poincaré developed a special technique to solve this type of equation by expanding into a series in powers of small parameters not only the function $C(\xi, \eta)$ itself, but also the independent variables. Let us introduce new variables:

$$z = \eta \qquad \alpha\tau = \xi - \beta f(\tau, z) \tag{32}$$

where $f(\tau, z)$ is some arbitrary function which will later be selected in such a manner as to permit the elimination of a dangerous singularity manifesting itself at the point $\xi = 0$.

Equations for $C^{(0)}$ and $C^{(1)}$ are obtained from Eq. (26) rewritten in new variables. Limiting ourselves to the first term of the series, we arrive at the equation for $C^{(0)}$:

$$\frac{\partial C^{(0)}}{\partial \tau} = \frac{\partial^2 C^{(0)}}{\partial z^2} \tag{33}$$

The solution of this equation is well known. That is, the solution of Eq. (33) with proper boundary conditions [Eq. (29)] can be written as:

$$C^{(0)}(\tau, z) = C_{\text{eq}}\,\text{erf}\left(\frac{z}{2\sqrt{\tau}}\right) \tag{34}$$

where erf, as always, is

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\, dt \tag{35}$$

The equation for $C^{(1)}$ can now be found by substituting for (34) in (31) and (28). In the first approximation (first order of the small parameter β), we have

$$\frac{\partial C^{(1)}}{\partial \tau} - \frac{\partial c^{(1)}}{\partial z^2} = \frac{C_{eq} \cdot z}{2\sqrt{\pi \tau^3}} \cdot \exp\left(-\frac{z^2}{4\tau}\right)$$
$$\cdot \left[z^2 + \frac{\partial f}{\partial \tau} - \frac{\partial^2 f}{\partial z^2} + \frac{\partial f}{\partial z}\left(\frac{z}{\tau} - \frac{2}{\tau}\right) - \left(\frac{3}{2\tau} + \frac{z^2}{4\tau^2}\right)\right] \tag{36}$$

Boundary conditions can be expressed as follows:

$$\left.\begin{aligned} &C^{(1)} \to -C_{eq}(4\pi\tau^3)^{-1/2} \lim_{z\to\infty}[zf(\tau, z)e^{-z^2/4\tau}]\ (\tau > 0) \\ &C^{(1)} \to -C_{eq}(4\pi\tau^3)^{-1/2} \lim_{z\to\infty}[zf(\tau, z)]\ (\tau > 0) \\ &C^{(1)} = 0\ (t = 0, z > 0) \end{aligned}\right\} \tag{37}$$

The analysis shows that the dangerous divergence in the derivative from $C^{(1)}$ in the limit $z \to 0$ ($1/z^{3/2}$ type) is determined by terms in Eq. (36), namely:

$$\frac{z^2}{4\tau^2} - \frac{3}{2\tau}$$

Let us select a function $f(\tau, z)$ in such a manner that it will eliminate the above terms from Eq. (36). In order to accomplish this, the function $f(\tau, z)$ should satisfy the equation:

$$\frac{\partial f}{\partial \tau} + \frac{\partial f}{\partial z}\left(\frac{z}{\tau} - \frac{2}{z}\right) - \frac{\partial^2 f}{\partial z^2} = \frac{3}{2\tau}\left(1 - \frac{z^2}{6\tau}\right) \tag{38}$$

Since Eq. (38) is a partial differential equation, the function $f(\tau, z)$ is not completely determined by this equation. Additional conditions (equivalent to boundary conditions) that could be imposed on the function $f(\tau, z)$ will be formulated a little later [see Eq. (43)]. Meanwhile, let us notice from the appearance of Eq. (38) that $f(\tau, z)$ is a self-similar function of variables τ and z. Assuming that $\zeta = z/\sqrt{\tau}$, we can obtain the equation for $f(\zeta)$ in total derivatives:

$$f''(\zeta) + \left(\frac{2}{\zeta} - \frac{\zeta}{z}\right) f'(\zeta) = \frac{\zeta^2}{4} - \frac{3}{2} \tag{39}$$

The general solution of the above equation can be expressed as follows:

$$f(\zeta) = -\frac{\zeta^2}{4}\frac{A_1}{\zeta} e^{\zeta^2/4}$$
$$\cdot \left[1 + \sum_{k=0}^{\infty} (-1)^{k+1} \frac{1}{2^{k+1}} \frac{\zeta^{2k+2}}{(k+1)!!}\right] + A_2 \tag{40}$$

where A_1 and A_2 are arbitrary constants. In order to determine A_1 and A_2 and therefore the function $f(\zeta)$, let us find the density of the diffusion flux at the liquid surface:

$$j(\xi) = D\left(\frac{\partial C}{\partial y}\right)_{y=h_0} \tag{41}$$

Lengthy but straightforward calculations show that $j(\xi)$ can be expressed by the Eq. (42) below with an accuracy of up to factors of $\sqrt{\text{Pe}}$ inclusive:

$$j(\xi) = \frac{DC_{eq}}{h_0}\frac{\sqrt{\alpha\,\text{Pe}}}{\pi\xi}\left\{1 - \frac{1}{C_{eq}\,\text{Pe}}\frac{\sqrt{\pi\xi}}{\alpha}\right.$$
$$\left.\cdot\left[\frac{\partial C^{(1)}}{\partial z} - \frac{C_{eq}}{2}\frac{\sqrt{\alpha^3}}{\pi\xi^3}\lim_{z\to 0}\left(z\frac{\partial f(\xi, z)}{\partial z}\right)\right]\right\} \tag{42}$$

The flux density j should remain integrable, i.e., without singularities of $1/\xi^{3/2}$ type.

In order to ensure that the above requirement is met, it is necessary for f to satisfy the equality

$$\lim_{z\to 0}\left[z\frac{\partial f(\xi, z)}{\partial z}\right] < \infty \qquad x > 0 \tag{43}$$

Condition (43) is the complete definition of the function $f(\xi, z)$ or $f(\zeta)$. It follows from expressions (40) and (42) that $A_1 = 0$. The constant A_2 is neither a term of the equation for $f(\xi, z)$ nor part of the boundary conditions. Therefore, it can be assumed that $A_2 = 0$ also. Finally,

$$f(\zeta) = -\frac{\zeta^2}{4} = -\frac{z^2}{4\tau} \tag{44}$$

As a result, after simple calculations we obtain

$$C^{(1)}(\tau, z) = \frac{C_{eq}z}{z}\frac{\sqrt{\tau}}{\pi} e^{-z^2/4\tau}\left(1 + \frac{z^2}{3\tau}\right) \tag{45}$$

Substituting for (44) and (45) in (42); the final expression for the diffusion flux density into the depth of film is derived:

$$j(\xi) = \frac{DC_{eq}}{h_0}\frac{\sqrt{\alpha\,\text{Pe}}}{\pi\xi}\left(1 - \frac{\beta\xi}{2\alpha^2}\right) \tag{46}$$

or, in dimensional form,

$$j = C_{eq}\frac{\sqrt{3D\bar{U}}}{2\pi x}\left[1 - \left(\frac{D}{2h_0\bar{U}}\right)\frac{L}{h_0}\right] \tag{47}$$

The full flux of impurities per unit time is determined by the integral (45).

Before discussing the above results, let us briefly address the assumptions employed.

The first assumption concerns the waveless flow regime. It is absolutely certain that the films flow down in the wave regime when Re > 20. However, it is reasonable to disregard surface waves for this preliminary type of analysis. As was proved by Brenner [5], the wave-type movement leads to the substitution of the diffusion coefficient by the effective diffusion coefficient, equal to:

$$D_{\text{eff}} = D\left[1 + \text{const}\left(\frac{a}{\lambda}\right)^2\right] \tag{48}$$

where a is the amplitude and λ is the wavelength. As long as the film is stable, λ is larger than a. The influence of the wave movement on the diffusion flux is proportional to $\sqrt{D_{\text{eff}}}$ and, therefore, is relatively small.

The second, less obvious but very important, assumption deserves special discussion. Therefore, Sec. 6 is dedicated to this subject.

6. STEAM FLOW/FILM INTERACTION

The interaction of film with the flow of steam is an important issue requiring further clarification. In principle, the fast-moving steam flow might either atomize a film or carry it along as a whole. Despite the fact that there is no precise quantitative theory regarding the liquid surface interaction with the turbulent flow of gas (steam), it is possible to formulate some qualitative considerations. Let us analyze an ideal situation of the flat liquid film interaction with the flow of gas, depicted in Fig. 2(a).

It will be shown in this section that the situation in which the gas is moving along the film is not very different from a situation in which the gas is flowing along a flat, solid surface. In the flow of gas flowing over the film surface, the laminar boundary layer is in the initial process of being established. This layer can be turbulized under some definite conditions. This turbulization can take place over the length L if the gas velocity is high enough. If, as shown below, the Reynolds number of the flow of gas over the liquid boundary layer does not differ substantially from that over the solid surface, turbulization of a laminar boundary layer can take place when the effective Reynolds number of this boundary layer exceeds the critical value:

$$\text{Re}_{\text{bound.layer}} \approx \sqrt{\text{Re}} \approx 1600$$

If, for example, $U_{\text{gas}} \sim 30$ m/s, $\nu_\ell \sim 0.1$ (for gas at standard conditions), then $L \sim 5$ m. Most probably the value of L is overestimated here, since in reality the film surface is mobile and rough due to the presence of capillary waves. Nevertheless, the boundary layer is turbulized, not instantaneously but most probably over a length L close to 1 m.

If the above is sufficiently accurate, it makes sense to analyze the laminar boundary layer of gas moving along the laminar, smooth, mobile film. The continuity of the tangential velocity and stress tensor components on the liquid surface gives us the boundary conditions:

$$V_g = V_\ell \qquad \mu_g\frac{\partial V_g}{\partial y} = \mu_\ell\frac{\partial V_\ell}{\partial y} \qquad \text{at } y = h \tag{49}$$

where h is the film thickness determined by the condition

$$\int_0^h V_\ell\,dy = Q \tag{50}$$

where Q is the total flow. Substituting for dimensionless parameters

$$U_g = \frac{V_g}{V_g^\infty} \qquad U_\ell = \frac{V_\ell}{\bar{U}} \tag{51}$$

where V_g^∞ is the gas velocity far from the interface and $\bar{U}$ is the average film velocity, we find:

$$U_g = \epsilon_1 U \qquad \frac{\partial U_g}{\partial y} = \epsilon_2\frac{\partial U_\ell}{\partial y} \qquad \text{at } y = h \tag{52}$$

$$\epsilon_1 = \frac{\bar{U}}{V_g^\infty} \qquad \epsilon_2 = \frac{\mu_g V_g}{\mu_\ell \bar{U}} \tag{53}$$

The dimensionless parameter ϵ_1 is obviously small compared to unity. It is interesting to note that the parameter ϵ_2 is also smaller than unity under the conditions under consideration. Assuming that $V_g^\infty/\bar{U} \simeq 30$ but that $\mu_g/\mu_\ell \simeq 2 \times 10^{-2}$, it can be derived that $\epsilon_2 \approx 0.6$. The latter has the following interpretation:

1. If $\epsilon_1 = 0$, the velocity distribution in gas coincides with the velocity distribution in gas overflowing the solid surface.
2. If $\epsilon_1 < 1$, it is possible to determine U_g by means of the method of consecutive approximations:

$$U_g = U_g^{(0)} + \epsilon_1 U_g^{(1)} + \epsilon_1^2 U_g^{(2)} + \dots \tag{54}$$

where $U_g^{(n)}$ is the correction to the velocity distribution $U_g^{(0)}$.

On the other hand, when $\epsilon_2 = 0$, the boundary condition on the film surface coincides with the condition for the free film flow [which leads to the velocity distribution (5)]. In this case the moving gas does not drag the film along.

If $\epsilon_2 \ll 1$, it is possible to write:

$$U_\ell = U_\ell^{(0)} + \epsilon_2 U_\ell^{(1)} + \epsilon_2^2 U_\ell^{(2)} + \dots \tag{55}$$

where $U_\ell^{(0)}$ coincides with (5). The latter thickness changes when the flow of gas drags the film along. It can be written by analogy with Eq. (55) that

$$h = h_0 + \epsilon_2 h^{(1)} + \dots \tag{56}$$

Correction factors $U_g^{(1)}$, $U_\ell^{(1)}$, and $h^{(1)}$ were determined after simple but lengthy calculations. What is important here is that if ϵ_1 and ϵ_2 are small, correction factors to the velocity distributions are also small. This means that the flow of gas is not capable of destroying the film integrity (at least in the case of laminar gas flow).

As a result of the calculations, the following equation is obtained for the film thickness:

$$h \simeq h_0 \left[1 \pm 0.3\, \epsilon_2 \left(\frac{U_g^\infty}{\bar{U}_\ell} \frac{x}{h_0} \right)^{1/2} \frac{1}{\sqrt{\mathrm{Re}_\ell}} \right] \tag{57}$$

where the $(-)$ sign is associated with the counterflow situation. The inequality $h \ll h_0$ means physically that the film can be atomized by the steam flow. Calculations based on Eq. (57) show, however, that the complete turbulization of the hydrodynamic boundary layer is reached before atomization takes place.

Let us now consider the possibility of rupture and atomization of a film by fully turbulent gas flow. For this purpose let us consider a pressure fluctuation p' in a turbulent gas flow. If $p' < 0$, the gas flow acts as if it sucked out roughnesses of effective size λ from the film. In this case, a capillary pressure of σ/λ is created on the film surface, where σ is the surface tension. If the inequality $p' > \sigma/\lambda$ is fulfilled, liquid destruction can take place. The maximum scale of the gas pressure fluctuations is determined as follows:

$$p' \approx \rho_g (V_g^\infty)^2$$

It is obvious that the most unfavorable scale for λ is $\lambda \approx h_0$. Therefore, the more favorable condition for film rupture is

$$\rho_g (V_g^\infty)^2 > \frac{\sigma}{h_0}$$

with $\rho_g \approx 10^{-5}$, $V_g^\infty \approx 30$ m/s, $\sigma \approx 40$ din/cm^2, $\rho_g (V_2^\infty)^2 \approx 10^2$, $\sigma/h_0 \approx 4 \times 10^2$. Although this estimate is very rough, it clearly demonstrates that film destruction at low-pressures is impossible even for substantial steam velocities. Film destruction can take place only under special circumstances associated, for example, with extremely high steam flow velocities or local film nonuniformities.

Overall, it is our impression that the film remains intact until it leaves the lowest tube. Of course, the latter statement should not be taken literally. Under nonuniform conditions as exist in actual condensers, a hydrodynamic situation could make it possible for the steam to encounter and break the film in some weak spots. This would trigger complete film destruction in weak areas. Therefore, it is reasonable to expect that in an operating condenser some film breaks and droplets will be found, especially in peripheral areas and steam flow clearances. However, it is still possible to conclude that generally film-type condensation prevails throughout the condenser. Film-type condensation is further assured by minimum deviations from the properly selected small ratio of a ligament to a tube diameter.

7. CONCLUSIONS AND RECOMMENDATIONS

Despite its simplicity (or perhaps because of it), Eq. (47) allows us to draw a number of important conclusions:

1. The first term in this equation corresponds to the situation where the liquid film thickness δ', which effectively resists the dissolving process (the diffusion boundary-layer thickness), is small compared to the depth of the liquid h_0. By definition,

$$\delta' = \frac{D\, \Delta C}{j} \simeq \frac{\sqrt{2\pi D x}}{3\bar{U}} \tag{58}$$

Due to a very small liquid diffusion coefficient D, the condition $\delta' \ll h_0$ is satisfied even for relatively long and thin films.

The second term in Eq. (47) accounts for a possible influence of the finite film thickness on the mass transfer process. This term is directly proportional to the small parameter (or inversely proportional to the Peclet number). Simple estimation shows that, despite the fact that this term includes the large numerical parameter L/h_0, it is still small compared to the first term. Therefore, on the basis of qualitative analysis, it is possible to conclude that the mass transfer rate is determined primarily by the film surface velocity. This is a significant conclusion, since it can be used to prove that if the slow stage of the air-dissolving process is the convective diffusion in the liquid film, the rate of the convective transfer process is the same as in a film moving with the same velocity as its surface. The mass flux is proportional to the average film velocity and increases with film thickness. Conversely, the dissolving process is not dependent on film thickness under a given flow-down velocity. Here a substantial difference between the dissolving and the heat transfer processes is found. In the heat transfer case the dimensionless Peclet number is

$$\text{Pe}_{\text{therm}} = \frac{UL}{\kappa} = \frac{UL}{\nu}\frac{\nu}{\kappa} = \text{Re} \cdot \text{Pr}_{\text{therm}}$$

where κ is the thermal conductivity coefficient and Pr_{therm} is the thermal Prandtl number. The Prandtl number is not too large for water at temperatures between 90 and 100°C. Therefore, in a film moving in a laminar regime, the temperature drops through the entire film thickness, that is, $\delta'_{\text{therm}} \approx k_0$. The film thickness increase (under a fixed-flow-rate condition) affects negatively the heat transfer process. Although this problem lies outside the scope of this work, it is necessary to point out that dropwise condensation appears to be more effective than film condensation. Furthermore, it appears reasonable to assume that some optimum distribution for the droplet and film condensations could be found for a given required condensation flow rate.

2. The fact that (under the assumptions of this study) the dissolving is determined primarily by the film surface velocity is important from one more viewpoint. Namely, in a first approximation it is known that, the convective diffusion process in a liquid is determined by its free surface velocity under a variety of essentially different conditions, such as film, film in a gas flow, droplet, etc. Therefore, it appears that the assumptions regarding film continuity and simplistic film flow-down geometry are not very important. Condensate, for example, could wander inside a tube bundle, or partially disintegrate into droplets. Nevertheless, the general character of the convective diffusion process will remain practically unchanged. A rate of dissolving could be estimated at least qualitatively in accordance with the first term of Eq. (47). However, it is important to determine not only the dissolving rate but, most important, the full amount of gas dissolved in the condensate or, more precisely, oxygen concentration in the condenser hot well.

The net mass of dissolved gas could be determined as:

$$J = \left(\int_0^L j\,dx\right) \cdot H \cdot N \approx \left(\sqrt{D\bar{U}L}\right) NHC_{\text{eq}} \quad (59)$$

where L = film traveling length, H = film width, and N = number of films. The volume of liquid reaching the hot well per unit of time is

$$V = \bar{U}h_0HN$$

The J/V ratio, which is of interest here, could be expressed as:

$$r = \frac{J}{V} \approx C_{\text{eq}}\frac{\sqrt{DL}}{h_0^2\bar{U}} \sim \left(\frac{\sqrt{D\nu L}}{g'h_0^4}\right)C_{\text{eq}} \quad (60)$$

This expression contains only one unknown, h_0. It is necessary to stress here that the expression obtained for the ratio r determines the upper limit of the gas concentration C in the hot well. The complete equation (47) for j gives smaller values of C. However, the order of magnitude of the latter remains the same. What is more important here is that impurity concentration could vary throughout the condenser.

Assuming reasonable values for $h_0 \approx 0.2$–0.3 cm and $L \approx 10^2$ cm, the following is obtained for the ratio r:

$$r \approx 10^{-3}C_{\text{eq}}(p_g)$$

In conclusion, we would like to formulate a problem which, in our opinion, is of substantial interest. Namely, some optimum situation characterized by maximum achievable condensation with minimum achievable oxygen absorption should exist. Indeed, the hot well oxygen concentration decreases with decreasing length L and increasing film thickness. On the other hand, the very same factors decrease condensation efficiency. No condensation theory for flowing films exists at present. The development of such a theory, first, is long overdue, and second, would be instrumental in finding the above-mentioned optimum values for condensation and air absorption.

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