MODELING OF PLANE ELECTRICAL FIELDS IN ELECTROLYTES BY MEANS OF INVERTED MODELS

A.I. Maslii, N.P. Poddubnyi,
and N.P. Gnusin

UDC 620.197.5

Modeling of the distribution of an electrical field is widely used when solving many important applied problems [1, 2]. Direct plotting of the lines of flow of the investigated model is difficult, although in principle it is possible, for example, by means of a rotating dual probe. In the meantime a method, called field inversion, is widely used in technology [3] which reduces the plotting of lines of flow to plotting of equipotential lines. Inverted models are also used for measuring the primary distribution of flow in a number of electrolytic cells [4, 5].

Underlying the inversion of plane electrical fields is the fact that the potential function V(m) and stream function ψ(m) are conjugate, i.e., satisfy the so-called Cauchy-Riemann conditions

$$\frac{\partial V}{\partial l}=\frac{\partial \psi}{\partial n}, \tag{1}$$

$$\frac{\partial V}{\partial n}=-\frac{\partial \psi}{\partial l}, \tag{2}$$

where l and n are two mutually perpendicular directions, rotation from l to k being accomplished counterclockwise. Fulfillment of conditions (1) and (2) means the equipotential lines V=const and lines of flow ψ = const are mutually orthogonal. Therefore, we can construct two geometrically similar fields, direct and inverse, the equipotential lines of the direct field becoming the lines of flow for the inverted field and vice versa. The following well-known principle [6] is commonly used for constructing the conjugate field ψ(m): if functions V and ψ satisfy conditions (1) and (2) at the boundaries of the region (l is tangential to the boundary, n is the inner normal), they are conjugate at any inner point. Therefore, the problem of constructing the conjugate fields amounts to inversion of the corresponding boundary conditions. We will consider the inversion of certain boundary conditions applicable to electrical fields in electrolytes.

1. Primary Distribution of Flow. In this case there are two types of boundaries in the electrical field: on the electrode V = const and on the insulator $\partial V/\partial n = 0$. Applying conditions (1) and (2) to them, we obtain, respectively, for the boundary conditions of the inverted field: $\partial\psi/\partial n = 0$ and ψ = const. Thus, to obtain the inverted field in this case it suffices that the boundaries of the field with the electrodes and with the insulator change places.

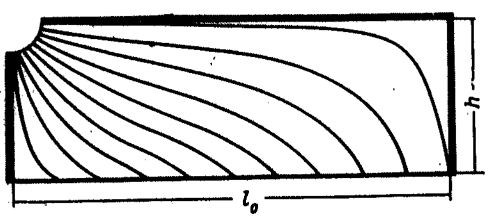

Fig. 1

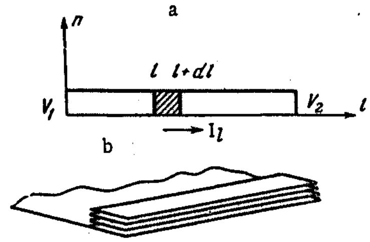

Fig. 2

Fig. 1. Pattern of the distribution of lines of flow in a limiting field of a slotted tank (l_0/h = 2.8) obtained on an inverse model.

Fig. 2. Diagram for deriving the relationship between potential and flow when assigning conditions conjugate with linear conditions in the inverse model.

Institute of Physicochemical Principles of Processing Mineral Raw Materials, Siberian Division, Academy of Sciences of the USSR. Translated from Zashchita Metallov, Vol. 5, No. 3, pp. 341-344, May-June, 1969. Original article submitted May 17, 1967.

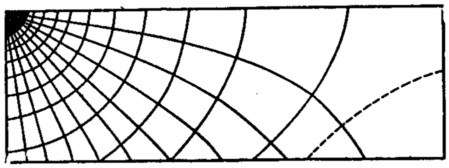

Fig. 3. Pattern of the electrical field in a slotted tank ($l_0/h = 2.8$) for linear conditions at the cathode ($\alpha/\rho_s l_0 = 0.6$).

2. Limiting Field. A limiting field is characterized ideally by a uniform distribution of flow at the electrode. In this case, maintaining the conditions on the insulator at the boundary with the electrode in field V, we have

$$D_N = -\frac{1}{\rho_s}\frac{\partial V}{\partial n} = \text{const}, \tag{3}$$

where D_N is the current strength per unit length of electrode and ρ_s is the surface resistance of the model's material. The condition conjugate to it in field ψ, obviously, is

$$\frac{1}{\rho_s}\frac{\partial \psi}{\partial l} = \text{const}, \tag{4}$$

which requires that the potential gradient along the electrode in the inverted field be constant. In principle this can only be when the current passing through the electrode does not flow out into the field. For the practical fulfillment of Eq. (4) it suffices that the resistance of the material along the electrode be much lower than that of the model's material. Figure 1 shows an example of plotting a limiting field on the inverted model of a slotted tank ($l_0/h = 2.8$; $l_0 = 20$ cm). The model was cut out of conducting paper. For assignment of condition (4) at the cathode we used a rolled Nichrome wire ($r = 4.5$ Ω/m) fastened to the boundary by means of India ink. The conductors were made of copper foil. The degree of fulfillment of condition (4), and hence of the correctness of the solution of the formulated problem, can be determined from Fig. 1, from where we see that all lines of flow rest on practically the same sections of the electrode.

3. Inversion of Electrical Fields of Polarized Electrodes. The boundary condition on the electrode for field V in this case is the equation

$$V = f(D_N) + \text{const}.* \tag{5}$$

Differentiating it with respect to l, we obtain

$$\frac{\partial V}{\partial l} = \frac{\partial V}{\partial D_N}\cdot\frac{\partial D_N}{\partial l}. \tag{6}$$

Using (2) and taking into account that

$$D_N = -\frac{1}{\rho_s}\frac{\partial V}{\partial n}, \quad a\ \frac{\partial V}{\partial D_N}$$

is none other than polarizability $\alpha(D_N)$, the boundary condition for the inverted field is obtained in the form

$$\frac{\partial \psi}{\partial n} = \alpha\left(\frac{1}{\rho_s}\frac{\partial \psi}{\partial l}\right)\cdot\frac{1}{\rho_s}\frac{\partial^2 \psi}{\partial l^2}. \tag{7}$$

Since in the general case α is a nonlinear function $[(1/\rho_s)(\partial\psi/\partial l)]$, realization of boundary condition (7) in the inverted model becomes extremely difficult.

In the particular case of the independence of α on $(1/\rho_s)(\partial\psi/\partial l)$ (case of linear polarization) condition (7) takes the form

$$\frac{\partial \psi}{\partial n} = \frac{\alpha}{\rho_s}\cdot\frac{\partial^2 \psi}{\partial l^2}. \tag{8}$$

We will show that the last conditions can be realized by means of a thin conducting plate applied to the boundary corresponding to the electrode, as shown in Fig. 2. We denote by r the resistance of the plate per unit length. Then the strength of the current flowing along the plate from the anode to the cathode is

$$I(l) = -\frac{1}{r}\frac{\partial \psi}{\partial l}. \tag{9}$$

* The arbitrariness of the constant in Eq. (5) is due to the fact that in conformity with actual practice, in carrying out the process of electroplating, we examine electrical fields at assigned current density and not voltage in the tank.

Examining the hatched unit section of the plate and applying to it Kirchhoff's law, we obtain

$$I(l+dl)-I(l)=D_N'(l)\cdot dl.$$

From here we obtain

$$D_N'(l)=\frac{\Delta I(l)}{dl}\simeq\frac{\partial I(l)}{\partial l}. \tag{10}$$

Using (9), we obtain

$$D_N'(l)=-\frac{1}{r}\frac{\partial^2\psi}{\partial l^2} \tag{11}$$

But D_N' in conformity with Ohm's law is equal to $-(1/\rho)(\partial\psi/\partial n)$ and finally we obtain

$$\frac{\partial\psi}{\partial n}=\frac{\rho_s}{r}\frac{\partial^2\psi}{\partial l^2}. \tag{12}$$

Comparing (12) and (8) we see that a thin conducting plate actually enables us to obtain the required boundary conditions. In so doing $\alpha=\rho_s^2/r$.

The prescribed conductivity of the plate can be obtained easily if for the plate we use a strip of the conducting material from which the model was made, preliminarily pleating it into several layers (Fig. 2b). Let the length of the strip be l_0 and the width be l. Then $r=\rho_s/l$, and the criterion of electrochemical similarity is

$$\frac{\alpha}{\rho_s l_0}=\frac{l}{l_0}.$$

It is curious to note that the same width of the boundary strip l, is required for assigning the linear boundary conditions on the direct model and the conditions conjugated with them on the inverted model [7].

Figure 3 shows the pattern of the lines of force and equipotential lines in a slotted tank ($l_0/h=2.8$) for equipotentiality of the anode and linear boundary conditions at the cathode ($\alpha/\rho_s l_0=0.6$). The pattern was obtained in two stages: at first on a direct model we measured, by the usual method, the equipotential lines and then on the inverted model the conjugate conditions were assigned by means of the "pleats" and the lines of flow obtained. The correctness of modeling was confirmed in this case by the orthogonality of the obtained equipotential lines and lines of flow.

LITERATURE CITED

1. Yu.Ya. Iossel', É.S. Kochanov, and M.G. Strunskii, Problems of the Calculation and Modeling of Electrochemical Corrosion Protection of Ships [in Russian], Sudostroenie (1965).
2. J. Euler and Z. Horn, Electrochim. Acta, 10, 1057 (1965).
3. P.F. Fil'chakov and V.N. Panchishin, Electrohydrodynamic Analogy Integrators. Modeling of Potential Fields on Conducting Paper [in Russian], Izd. AN UkrSSR, Kiev (1961).
4. G.F. Kinney and G.V. Festa, Plating, 41, 380-387 (1954).
5. R. Rousselot, J. Rech. Centre Nat. Rech. Scient., 11, 231 (1960); Metal Finish, 57, 56 (1959).
6. M.A. Lavrent'ev and B.V. Shabat, Methods of the Theory of Functions of Complex Variables [in Russian], Fizmatgiz, Moscow (1958).
7. N.P. Gnusin and A.I. Maslii, in: Electrical Fields in Electrolytes [in Russian], Nauka, Novosibirsk (1967), p. 60.