DISTRIBUTION OF THE PROTECTIVE CURRENT OF A DISK PROTECTOR WITH A RING INSULATING SHIELD

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UDC 620.197.5

Disk protectors are among the most typical elements of electrochemical protection systems used for combatting corrosion of various metal structures. To achieve the necessary efficiency of such protectors, ring insulating shields (reinforced organic coating, rubber gluing, etc.) are usually placed near them. With rational selection of the size of the shield it is possible to increase noticeably the zone of protective action of the disk protector and to eliminate the possibility of overprotection in its immediate vicinity.

The optimal size of a ring insulating shield can be determined only on the basis of an analysis of the protective current in the system in question, which in the simplest case (Fig. 1) is an infinitely extended flat surface of the metal being protected 1, an infinitely thin flat disk protector 2, and a ring insulating shield 3 separating them.

Let us consider the primary distribution of the current on an unpainted surface 1, which is known to have a greater inhomogeneity than the true distribution of a protective current with consideration of polarization and painting of the surface 1. When calculating the primary distribution of the protective current in the examined system, the potential of the infinitely extended protected surface should be taken to be equal to zero, and the potential of the protector to be equal to the difference of the steady potentials of the cathodic and anodic processes (C). Considering, in addition, the shield to be completely nonconductive, we can easily establish that with respect to formulation, the problem in question is similar to the problem of calculating the density of a charge applied on an infinitely extended grounded metal plane with a round cut-out in the field of a disk electrode 2 whose potential is equal to C.* In turn, the latter problem can be solved by using G. A. Grinberg's well-known theorems on the distribution of a charge induced on thin open shells [1]. For a degenerated shell in the form of a plane with a round cut-out situated in an arbitrary axisymmetric field, this solution has the form [2]

$$\sigma_1(r)+\sigma_2(r)=\frac{4\varepsilon}{\pi}\frac{d}{dr}\int_R^r\frac{r\,dS}{\sqrt{r^2-S^2}}\times\frac{d}{dS}\int_S^\infty\frac{SV_0(t)\,dt}{t\sqrt{t^2-S^2}}, \tag{1}$$

$$\sigma_1(r)-\sigma_2(r)=2\pi\varepsilon E_z^0(r), \tag{2}$$

* In the given case we can disregard the effect of surface 1 on the potential of surface 2.

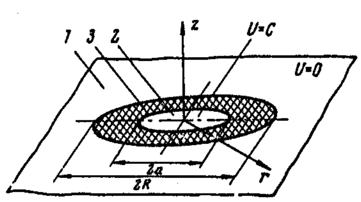

Fig. 1

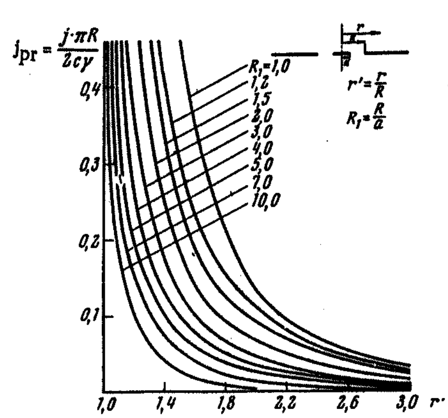

Fig. 2

Translated from Zashchita Metallov, Vol. 5, No. 2, pp. 243-246, March-April, 1969. Original article submitted May 24, 1968.

where σ_1 (r) and σ_2 (r) are the charge density on the upper and lower sides of the shell; R is the radius of the round cut-out; r is a coordinate reckoned from the center of the cut-out; $V_0(z)$ and E_Z^0 (r) are the potential and normal component of the external field at an examined point of the plane; ε is the dielectric constant of the external medium. In the examined case the function $V_0(r)$ represents the potential of the electrostatic field of an isolated disk electrode 2 and is expressed by the known formula [3]

$$V_0(r_1)'\Big|_{\substack{z_1=0\\ r_1>1}} = C\cdot\arcsin\frac{1}{r_1}, \tag{3}$$

where $r_1 = r/a$; $z_1 = z/a$; C and a are respectively the potential and radius of the electrode. By virtue of symmetry of this field relative to plane z = O the charge densities on each of its sides are the same ($\sigma_1(r) = \sigma_2(r)$) and can be found directly from formula (1). Using the analogy between static and steady electric fields, we find the formula for the density of the protective current:

$$j(r_1)\Big|_{r_1\geqslant R_1>1} = \frac{2C\gamma}{\pi a}\cdot\frac{d}{dr_1}\int\limits_{R_1}^{r_1}\frac{r_1\cdot dS_1}{\sqrt{r_1^2-S_1^2}}\cdot\frac{d}{dS_1}\int\limits_{S_1}^{\infty}\frac{S_1\arcsin\frac{1}{t_1}}{t_1\sqrt{t_1^2-S_1^2}}dt_1, \tag{4}$$

where $R_1 = R/a$ is the ratio of the outside and inside radii of the ring shield; γ is the conductivity of the corrosion medium; S_1 and t_1 are dimensionless variables.

Performing the necessary calculations in this formula (see Appendix), we find that the primary distribution of the protective current of a disk protector with a ring insulating shield is determined by the expression

$$j(r_1)\Big|_{\substack{z_1=0\\ r_1\geqslant R_1>1}} = \frac{C\gamma}{\pi a}\left[\frac{1}{\sqrt{r_1^2-1}}\cdot\ln\frac{-r_1^2+\left(\sqrt{r_1^2-1}-\frac{R_1}{r_1+\sqrt{r_1^2-R_1^2}}\right)^2}{r_1^2-\left(\sqrt{r_1^2-1}+\frac{R_1}{r_1+\sqrt{r_1^2-R_1^2}}\right)^2}\right.$$
$$\left.+\frac{1}{\sqrt{r_1^2-R_1^2}}\cdot\ln\frac{R_1-1}{R_1+1}\right]. \tag{5}$$

The results of numerical calculations by this formula, presented in Fig. 2, agree well with the data of calculations of the primary distribution of the protective current of an unshielded disk protector ($R_1 = 1$), which is given by the known expression [4]

$$j(r_1)\Big|_{r_1\geqslant R_1=1} = \frac{2C\gamma}{\pi a}\cdot\frac{1}{r_1}\left[K\left(\frac{1}{r_1}\right)-\frac{1}{1-\left(\frac{1}{r_1}\right)^2}E\left(\frac{1}{r_1}\right)\right], \tag{6}$$

where $K(1/r_1)$ and $E(1/r_1)$ are complete elliptical integrals of the first and second kinds.

In conclusion we note that when using the method of successive approximations formula (5) can be regarded as the first approximation when considering polarization of the protected metal or the presence on it of any surface films of finite conductivity.

APPENDIX

For calculation of quadratures in formula (4) we examine at first the integral

$$I_1 = \int\limits_{S_1}^{\infty}\frac{\arcsin\frac{1}{t_1}\,dt_1}{t_1\cdot\sqrt{t_1^2-S_1^2}}.$$

Using the substitution $1/t_1 = \sin\alpha$ and then integrating by parts, we find [5]:

$$I_1 = \int\limits_0^{\arcsin\frac{1}{S_1}}\frac{\alpha\cdot\cos\alpha}{\sqrt{1-S_1^2\cdot\sin^2\alpha}}\cdot d\alpha = \frac{\pi}{2S_1}\arcsin\frac{1}{S_1}-\frac{1}{S_1}\int\limits_0^{\arcsin\frac{1}{S_1}}\arcsin(S_1\cdot\sin\alpha)\,d\alpha.$$

Then

$$\frac{d}{dS_1}(S_1 I_1) = -\int\limits_0^{\arcsin\frac{1}{S_1}} \frac{\sin\alpha}{\sqrt{1 - S^2\cdot\sin^2\alpha}}\,d\alpha = \frac{1}{2S_1}\ln(S_1^2 - 1) - \frac{1}{S_1}\ln(S_1 + 1).$$

Thus, expression (4) can be presented in the form

$$j(r_1) = \frac{2C\gamma}{\pi a}\left[\frac{1}{2}\cdot\frac{d}{dr_1}(r_1 I_2) - \frac{d}{dr_1}(r_1 I_3)\right], \tag{4a}$$

where

$$I_2 = \int\limits_{R_1}^{r_1} \frac{\ln(S_1^2 - 1)\,dS_1}{S_1\sqrt{r_1^2 - S_1^2}}, \qquad I_3 = \int\limits_{R_1}^{r_1} \frac{\ln(S_1 + 1)}{S_1\sqrt{r_1^2 - S_1^2}}\,dS_1.$$

Using the substitution $S_1 = r_1\cdot\sin\beta$, we find that

$$r_1 I_2 = \int\limits_{\arcsin\frac{R_1}{r_1}}^{\frac{\pi}{2}} \frac{\ln(r_1^2\cdot\sin^2\beta - 1)}{\sin\beta}\,d\beta.$$

When $R_1 > 1$ this integral can be differentiated with respect of r_1, as a result of which we find

$$\frac{d}{dr_1}(r_1 I_2) = 2r_1 \int\limits_{\arcsin\frac{R_1}{r_1}}^{\frac{\pi}{2}} \frac{\sin\beta}{r_1^2\cdot\sin^2\beta - 1}\,d\beta + \frac{1}{\sqrt{r_1^2 - R_1^2}}\ln(R_1^2 - 1),$$

or after integration

$$\frac{d}{dr_1}(r_1 I_2) = \frac{1}{\sqrt{r_1^2 - 1}}\ln\left[\frac{(r_1 - \sqrt{r_1^2 - 1})^2 - 1}{(r_1 + \sqrt{r_1^2 - 1})^2 - 1}\cdot\frac{(r_1 + \sqrt{r_1^2 - 1})^2 - \dfrac{R_1^2}{(r_1 + \sqrt{r_1^2 - R_1^2})^2}}{(r_1 - \sqrt{r_1^2 - 1})^2 - \dfrac{R_1^2}{(r_1 + \sqrt{r_1^2 - R_1^2})^2}}\right] + \frac{1}{\sqrt{r_1^2 - R_1^2}}\ln(R_1^2 - 1).$$

Likewise we obtain

$$\frac{d}{dr_1}(r_1 I_3) = \frac{1}{\sqrt{r_1^2 - 1}}\ln\left[\frac{1 + r_1 - \sqrt{r_1^2 - 1}}{1 + r_1 + \sqrt{r_1^2 - 1}}\cdot\frac{\dfrac{R_1}{r_1 + \sqrt{r_1^2 - R_1^2}} + r_1 + \sqrt{r_1^2 - 1}}{\dfrac{R_1}{r_1 + \sqrt{r_1^2 - R_1^2}} + r_1 - \sqrt{r_1^2 - 1}}\right] + \frac{1}{\sqrt{r_1^2 - R_1^2}}\ln(R_1 + 1).$$

Having substituted the last two expressions into formula (4a) and performing simple transformations, we arrive at formula (5) presented in the article.

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