CERTAIN PROBLEMS IN ESTIMATING CONTACT CORROSION OF PLANE AND CYLINDRICAL METAL SURFACES

(UDC 620.193.7)

Yu. Ya. Iossel', É. S. Kochanov, and M. G. Strunskii

The A. N. Krylov Central Scientific Research Institute
Translated from Zashchita Metallov, Vol. 1, No. 4,
pp. 410-419, July-August, 1965
Original article submitted January 18, 1965

Equipment and apparatus used in aggressive media usually have a large number of parts and assemblies made of different metals and alloys. Hence, in addition to the general corrosion of each of the parts, intense processes of contact corrosion occur which must be reckoned with when planning systems of anticorrosion protection. From this point of view the development of methods of estimating the expected contact corrosion of equipment and apparatus being planned acquires major importance.

The problem of estimating the contact corrosion of any piece of equipment or apparatus reduces to estimating the distribution of potential or current in the corroding surface. If the terminology of electrotechnics is employed, then such a problem is equivalent to estimating the stationary electric field under conditions of contact corrosion. Therefore, the posing of these problems does not differ in the least from the posing of any other problems in the theory of the electromagnetic field, i.e., one must formulate the equations and boundary conditions which the potential and current density of contact corrosion satisfy. If one limits oneself to finding the potential, which is usually more easily determined than the current density, then the problem reduces to the solution of the Laplace equation with boundary conditions specified by the shape of the corroding surfaces and the character of the electrochemical processes occurring on them. As is known [1], in the most general case this boundary condition has the form:

$$U_{M} - U = U_0 + \eta(j), \tag{1}$$

where U_M is the potential of the inner (next to the metal) face of the double layer; U is the potential of the outer face of the double electric layer; U_0 is the stationary electrode potential of the metal; $\eta(j)$ is a function characterizing the dependence of the overvoltage on the current density.

This dependence is expressed [1] by a nonlinear function which is determined by the mechanism of the corresponding electrochemical process. Therefore, in a strict formulation, the problem under consideration reduces to integrating the Laplace equation with nonlinear boundary conditions. An effective method of solving such problems does not exist at the present time; therefore, it becomes necessary to approximate the nonlinear boundary conditions given above (1) with some type of linear boundary conditions.

The most natural means of linearizing the boundary condition (1) is based on the piecewise-linear approximation of the polarization curves of the metals in contact. Such an approximation, for example, in the case of a typical cathode polarization curve for each of the separate sections leads to the boundary conditions:

$$U - b_i\gamma \frac{\partial U}{\partial n} = U_{M} - U_0, \tag{2}$$

where b_i is the specific polarizability of the metal for the chosen section of the polarization curve; γ is the specific electric conductivity of the corrosive medium; and n is the normal to the metal surface.

The distribution of potential and current density of contact corrosion found with such approximate boundary conditions is usually called secondary. In the particular case when the specific polarizability may be assumed close to zero, the boundary condition (2) takes the simpler form:

$$U = U_M - U_0, \tag{3}$$

corresponding to so-called primary distribution of potential and current.

It must be emphasized that the consideration of primary distribution of potential and current density by no means implies that polarization is neglected, but is rather merely the simplest and most convenient method of prescribing boundary conditions for an upper estimate of the maximum real values of potential and current density of contact corrosion.

Thus, in the approximate formulation of the problem, the estimation of contact corrosion can be reduced to finding the primary or secondary distribution of potential and current density characterizing this process.

As already mentioned, the form of the boundary conditions in problems of estimating contact corrosion, and hence the degree of complexity of their solution, is determined not only by the conditions of electrochemical character considered above but also by the shape of the surface of the metals in contact. Two of the simplest and most common shapes of such surfaces—plane and cylindrical—are considered below.

Let us consider the problem of computing the potential and current density characterizing the process of contact corrosion of a semi-infinite plane surface of an arbitrary metal in contact with the same type surface of another metal (Fig. 1, a). Of course, in actual equipment and apparatus there can be no semi-infinite surfaces; however, if they are of sufficiently large size they can be assumed semi-infinite with an accuracy which is entirely acceptable. The approximation to surfaces of small curvature often found in practice by the plane surfaces which we consider is also justified in the same measure.

Let us first assume that the surface of the metals in contact is completely free of any type of film. In this case the computation of the primary distribution of the current of contact corrosion in the system under consideration leads to integration of the Laplace equation with the following boundary conditions:

$$U_1|_{y=0} = \begin{cases} \dfrac{U_{02} - U_{01}}{2} & \text{if} \quad x > 0 \\ -\dfrac{U_{02} - U_{01}}{2} & \text{if} \quad x < 0, \end{cases} \tag{4}$$

where U_{02} and U_{01} are the stationary potentials of the metals in contact.

The solution of this problem is well known (see, for example, [2]) and has the form:

$$U_1 = \frac{U_{02} - U_{01}}{\pi} \operatorname{arctg} \frac{x}{y}. \tag{5}$$

Using this expression we immediately find the primary distribution of current density of contact corrosion we are interested in:

$$j_{y=0} = -\gamma \frac{\partial U_1}{\partial y}\Big|_{y=0} = \gamma \frac{U_{02} - U_{01}}{\pi} \frac{1}{x} \tag{6}$$

This elementary formula makes it possible to easily find the current density of contact corrosion at any point of the metal surface except the point $x = 0$ (the boundary of the dissimilar metals) where, because of the assumptions made, the current density becomes infinitely large. To remove this limitation let us now consider the primary distribution of potential and current of contact corrosion of metallic surfaces with the same shape having some kind of film of known electrical conductivity.* As is easily shown, the boundary conditions of such a problem have the form:

*It is easy to show [4] that in such a formulation the problem is equivalent to the computation of the distribution of potential and current of contact corrosion in the particular case when the specific polarizabilities of the metals in contact are taken equal to one another.

$$\left(U_2-\rho_{\text{кр}}\gamma\frac{\partial U_2}{\partial y}\right)_{y=0}=\begin{cases}\dfrac{U_{02}-U_{01}}{2} & \text{if} \quad x>0\\[2ex] -\dfrac{U_{02}-U_{01}}{2} & \text{if} \quad x<0,\end{cases} \tag{7}$$

where ρ_{kp} is the specific (per unit surface) resistance of the film layer (paint, say, to be specific).

In [3] and [4] it is shown that the approximate solution of such a problem can be obtained by a simple replacement of the y coordinate in formula (5) by the quantity $y + \rho_{kp}\gamma$. In this case it is easy to verify that the potential of contact corrosion is given by the formula:

$$U_2|_{y=0}=\frac{U_{02}-U_{01}}{\pi}\operatorname{arctg}\frac{x}{\rho_{\text{кр}}\gamma}, \tag{8}$$

while the components of current density are:

$$j_x|_{y=0}=\frac{U_{02}-U_{01}}{\pi}\frac{\rho_{\text{кр}}\gamma^2}{x^2+(\rho_{\text{кр}}\gamma)^2} \tag{9}$$

$$j_y|_{y=0}=\frac{U_{02}-U_{01}}{\pi}\gamma\frac{x}{x^2+(\rho_{\text{кр}}\gamma)^2}. \tag{10}$$

The modulus of the total vector of current density of contact corrosion, which determines the rate of dissolution of the metal, is given by the expression:

$$j|_{y=0}=\frac{U_{02}-U_{01}}{\pi}\frac{\gamma}{\sqrt{x^2+(\rho_{\text{кр}}\gamma)^2}} \tag{11}$$

The curves of primary distribution of current density of contact corrosion for a metal with a film and without a film in the system considered are given in Fig. 2. From these graphs it is evident, in particular, that as the distance from the boundary of contact of the dissimilar metals increases (with $x \gg \rho_{kp}\gamma$), the current densities of corrosion of the insulated and noninsulated metal become quite close. At small distances from the contact boundary the difference between them, as well as the error of the approximate formulas (8)-(11) obtained above, is very considerable. Therefore, it is also necessary to obtain a precise solution for the potential satisfying boundary conditions (7). To find this solution let us introduce the function V, related to the potential interesting us by the equation:

$$V=U_2-\rho_{\text{кр}}\gamma\frac{\partial U_2}{\partial y} \tag{12}$$

This function, just as U_1, satisfies the Laplace equation with boundary conditions (4) which means that it is determined by the expression (5), i.e.

$$U_2-\rho_{\text{кр}}\gamma\frac{\partial U_2}{\partial y}=\frac{U_{02}-U_{01}}{\pi}\operatorname{arctg}\frac{x}{y} \tag{13}$$

Inasmuch as this differential equation contains a partial derivative in only one variable it can be considered an ordinary differential equation containing the other variable as a parameter, i.e.

$$U_2-\rho_{\text{кр}}\gamma\frac{dU_2}{dy}=\frac{U_{02}-U_{01}}{\pi}\operatorname{arctg}\frac{x}{y}. \tag{14}$$

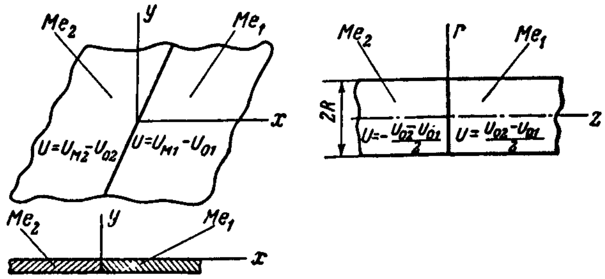


Fig. 1. Calculated models for determining the current density of contact corrosion of plane (a) and cylindrical (b) sections of the metal surface.

Integrating this differential equation and then putting y = 0, we find that the potential of contact corrosion of the metal is equal to:

$$U_2|_{y=0} = \frac{U_{02} - U_{01}}{\pi}\left[\frac{\pi}{2} + \operatorname{ci}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\sin\left(\frac{x}{\rho_{\text{кр}}\gamma}\right) + \operatorname{si}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\cos\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\right], \tag{15}$$

where $\operatorname{si}(t) = -\int\limits_t^\infty \frac{\sin x}{x}\,dx$ is the integral sine;

$\operatorname{ci}(t) = \int\limits_t^\infty \frac{\cos x}{x}\,dx$ is the integral cosine.

Differentiating expression (15) with respect to x and making use of formula (7), we arrive at the following expression for the components of current density of contact corrosion:

$$j_x|_{y=0} = +\frac{U_{02} - U_{01}}{\pi\rho_{\text{кр}}}\left[\cos\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\operatorname{ci}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right) - \sin\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\operatorname{si}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\right] \tag{16}$$

$$j_y|_{y=0} = -\frac{U_{02} - U_{01}}{\pi\rho_{\text{кр}}}\left[\sin\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\operatorname{ci}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right) + \cos\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\operatorname{si}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)\right] \tag{17}$$

The modulus of the total vector of current density is hence given by:

$$j|_{y=0} = \frac{U_{02} - U_{01}}{\pi\rho_{\text{кр}}}\sqrt{\operatorname{si}^2\left(\frac{x}{\rho_{\text{кр}}\gamma}\right) + \operatorname{ci}^2\left(\frac{x}{\rho_{\text{кр}}\gamma}\right)} \tag{18}$$

As is apparent, the precise expression obtained is relatively simple, and if use if made of tables of the integral sine and cosine [5] it can be easily used in engineering calculations.

Example. Let us determine the current density of contact corrosion at some point on the surface of an extended plane sheet of carbon steel situated in the ground with a specific electric conductivity of 0.02 $\Omega^{-1}\cdot m^{-1}$ in contact with a bronze sheet under the condition that both sheets are covered with a layer of paint with a specific resistance of $\rho_{kp} = 10^3\ \Omega\cdot m^2$.

The difference in the stationary electrode potentials of the metals in contact can be taken close to 0.3 V. Then assuming the distance of the point from the contact boundary to be x = 1 m, we find:

$$\operatorname{si}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right) = -1.52; \quad \operatorname{ci}\left(\frac{x}{\rho_{\text{кр}}\gamma}\right) = 2.42$$

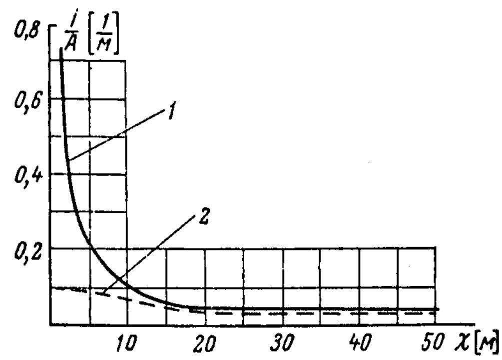


Fig. 2. Results of computing the distribution of current of contact corrosion on unpainted (1) and painted (2) plane sections of the metal surface:

$$A = \frac{U_{02} - U_{01}}{\pi}\gamma\left[\frac{a}{м}\right]$$

$$\rho_{кр}\cdot\gamma = 10\ [м]$$

and the density of corrosion current at the point considered is according to (18) equal to:

$$i\,|_{x=1м} = \frac{0.3}{3.14}10^{-3}\sqrt{(1.52)^2 + (2.42)^2} = 0.27\ \mathrm{mA/m^2}$$

Passing away from the contact boundary the density of the corrosion current will decrease correspondingly as follows from the formula used.

In considering the contact corrosion of a plane metal surface, cases in which there is a continuous, completely nonconducting film on particular parts of the surface are also of major interest. The problem of the primary distribution of current of contact corrosion can in this case be solved by the method of complex potential. Without presenting here the solution itself, we shall indicate the final formulas thus obtained for the current density of contact corrosion. These formulas are given in the table.

Let us now consider the case of contact corrosion in a system of two infinite metal surfaces of cylindrical shape (Fig. 1, b).

The problem of determining the primary distribution of the current of contact corrosion in this system reduces to integration of the Laplace equation written in cylindrical coordinates under the condition of axial symmetry ($\partial U/\partial\varphi = 0$) and with the boundary conditions:

$$\left.\begin{aligned} U\,|_{\substack{r=R\\ z>0}} &= \frac{U_{02} - U_{01}}{2} \\ U\,|_{\substack{r=R\\ z<0}} &= -\frac{U_{02} - U_{01}}{2} \end{aligned}\right\} \tag{19}$$

We will solve the problem by the method of the Fourier sine transform. The potential can then be represented in the form:

$$U = \int_0^\infty A(p)K_0(pr)\sin pz\,dp, \tag{20}$$

where A(p) is an unknown coefficient to be determined from the boundary conditions; $K_0(pr)$ is the MacDonald function of zero order.

To determine the constant A(p) let us insert the first of the boundary conditions (19) in formula (20) and compare the equation thus obtained with the known expression:

$$\int_0^\infty \frac{\sin pz}{p}dp = \frac{\pi}{2} \quad \text{with} \quad z > 0$$

The potential is then determined by the formula:

$$U = \frac{U_{02} - U_{01}}{\pi}\int_0^\infty \frac{K_0(pr)}{pK_0(pR)}\sin pz\,dp \tag{21}$$

The precise expression of the integral occurring in this formula has not yet been found. We therefore construct an approximate solution using the asymptotic representation of the MacDonald functions for large values of the argument [7]:

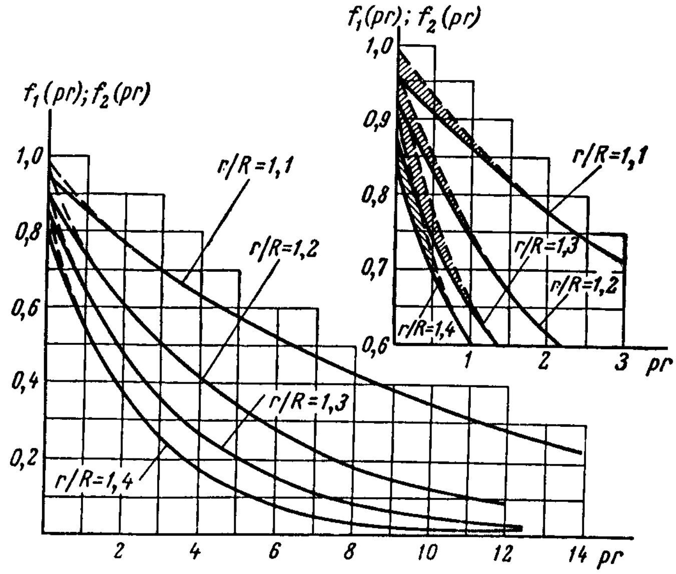


Fig. 3. Graphs of the exact and approximate values of the functions occurring in formula (21)

$$f_1(pr) = \frac{K_0(pr)}{K_0(pR)}$$

$$f_2(pr) \simeq \sqrt{\frac{R}{2}}\, e^{-pr(1-R/r)}$$

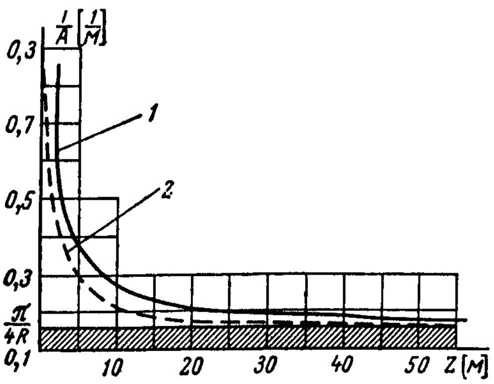


Fig. 4. Results of calculating the distribution of current of contact corrosion on unpainted (1) and painted (2) cylindrical sections of a metal surface:

$$A = \frac{U_{02} - U_{01}}{\pi}\gamma\left[\frac{a}{м}\right]$$

$$\rho_{кр}\cdot\gamma = 1\ [м]$$

$$R \simeq 5\ [м]$$

$$K(x) \approx \sqrt{\frac{\pi}{2x}}\, e^{-x}$$

Formula (21) then assumes the form:

$$U \approx \frac{U_{02} - U_{01}}{\pi}\sqrt{\frac{R}{r}}\ \mathrm{arc\ tg}\ \frac{z}{r-R} \qquad (22)$$

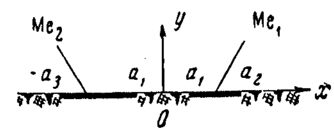
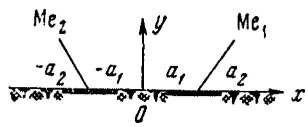
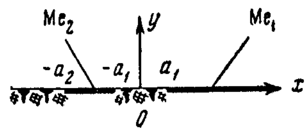
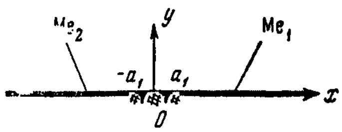
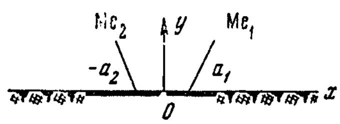
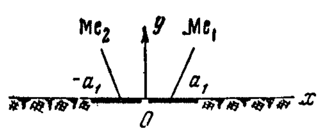
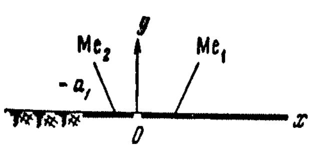
The error of the approximation can be evaluated with the help of the graph shown in Fig. 3 from which it is evident that even with r/R values close to unity the error does not exceed 1%

Differentiating (22) with respect to r, we obtain the following approximate expression for the primary distribution of current density of contact corrosion:

$$j_r\Big|_{\substack{r=R\\ z>0}} = \frac{U_{02} - U_{01}}{\pi}\gamma\left(\frac{\pi}{4R} + \frac{1}{z}\right) \qquad (23)$$

Making use of this expression, let us consider in conclusion the case of contact corrosion of the metallic surfaces studied under the condition that they are coated with a film of known electric conductivity. Such a problem reduces to integrating the Laplace equation with the boundary conditions: (see top of page 369)

Results of Computing the Current Density of Contact Corrosion of a Plane Metal Surface with a Nonconducting Film Deposited on Particular Sections of it

<table>
<tr><th>Type of computation model*</th><th>The real situation to which it refers</th></tr>
<tr><td>
</td><td>Contact corrosion of two dissimilar metallic sections of finite width separated by a section with a nonconducting film</td></tr>
<tr><td>
</td><td>The same with equal width of the uninsulated sections</td></tr>
<tr><td>
</td><td>The same with an infinite right uninsulated section</td></tr>
<tr><td>
</td><td>The same with both uninsulated sections infinite</td></tr>
<tr><td>
</td><td>Direct contact of two dissimilar metallic sections of finite width</td></tr>
<tr><td>
</td><td>The same with equal width of the uninsulated sections</td></tr>
<tr><td>
</td><td>The same with an infinite right uninsulated section</td></tr>
</table>

Reduced value of the current density of contact corrosion $j_{red} = \frac{j}{A\gamma\,(U_{02} - U_{01})}$		Value of the coefficient† A
for the left uninsulated section	for the right uninsulated section	
$\frac{1}{\sqrt{(x^2 - a_1^2)(a_2 + x)(a_3 - x)}}$	$\frac{1}{\sqrt{(x^2 - a_1^2)(a_2 - x)(a_3 + x)}}$	$\frac{\sqrt{(a_1 + a_2)(a_1 + a_3)}}{2K(k)}$, where $k = \sqrt{\frac{2a_1(a_2 + a_3)}{(a_1 + a_2)(a_1 + a_3)}}$
$\frac{1}{\sqrt{(x^2 - a_1^2)(a_2^2 - x^2)}}$	$\frac{1}{\sqrt{(x^2 - a_1^2)(a_2^2 - x^2)}}$	$\frac{a_2}{2K(k_1)}$ where $k_1 = \frac{a_1}{a_2}$
$\frac{1}{\sqrt{(x^2 - a_1^2)(a_2 - x)}}$	$\frac{1}{\sqrt{(x^2 - a_1^2)(x + a_2)}}$	$\frac{\sqrt{a_1 + a_2}}{2K(k_2)}$ where $k_2 = \sqrt{\frac{2a_1}{a_1 + a_2}}$,
$\frac{1}{\sqrt{x^2 - a_1^2}}$	$\frac{1}{\sqrt{x^2 - a_1^2}}$	$\frac{1}{\pi}$
$\frac{1}{x\sqrt{(a_1 + x)(a_2 - x)}}$	$\frac{1}{x\sqrt{(a_1 - x)(a_2 + x)}}$	$\frac{\sqrt{a_1 a_2}}{\pi}$
$\frac{1}{x\sqrt{a_1^2 - x^2}}$	$\frac{1}{x\sqrt{a_1^2 - x^2}}$	$\frac{a_1}{\pi}$
$\frac{1}{x\sqrt{a_1 - x}}$	$\frac{1}{x\sqrt{a_1 + x}}$	$\frac{\sqrt{a_1}}{\pi}$

*The heavy line indicates the uninsulated sections of the metal; the crosshatching indicates the sections with nonconducting film.

†The function K(k) entering the formula for A is the complete elliptic integral of type I with modulus k. The values of K(k) are tabulated in detail (see, for example, [6]) and can therefore be easily found for any value of the modulus k.

$$U-\rho_{\text{кр}}\gamma\frac{\partial U}{\partial r}=\begin{cases}\dfrac{U_{02}-U_{01}}{2} & \text{if} \quad r=R \text{ and } z>0\\[2ex] -\dfrac{U_{02}-U_{01}}{2} & \text{if} \quad r=R \text{ and } z<0\end{cases} \tag{24}$$

The approximate solution of this problem we again obtain with the help of the properties of the secondary distribution of potential considered in [3, 4] on the basis of which it is sufficient to replace the parameter R in (22) by the quantity $R-\rho_{\text{kp}}\gamma$ in order to find the solution.

Proceeding in such a manner, we find:

$$U\approx\frac{U_{02}-U_{01}}{\pi}\sqrt{\frac{R-\rho_{\text{кр}}\gamma}{r}}\operatorname{arctg}\frac{z}{r-R+\rho_{\text{кр}}\gamma} \tag{25}$$

Putting $r = R$ here, we find the value of the potential of contact corrosion in the form:

$$U|_{r=R}\approx\frac{U_{02}-U_{01}}{\pi}\sqrt{1-\frac{\rho_{\text{кр}}\gamma}{R}}\operatorname{arctg}\frac{z}{\rho_{\text{кр}}\gamma} \tag{26}$$

To find the components of the current density of contact corrosion, we differentiate expression (25) with respect to r and z and them put $r = R$. We obtain the result:

$$\begin{aligned} i_r|_{r=R}&=\gamma\frac{U_{02}-U_{01}}{\pi}\sqrt{1-\frac{\rho_{\text{кр}}\gamma}{R}}\left[\frac{1}{2R}\operatorname{arctg}\frac{z}{\rho_{\text{кр}}\gamma}+\frac{z}{z^2+\rho_{\text{кр}}^2\gamma^2}\right]\\ i_z|_{r=R}&=\gamma^2\frac{U_{02}-U_{01}}{\pi}\rho_{\text{кр}}\sqrt{1-\frac{\rho_{\text{кр}}\gamma}{R}}\cdot\frac{1}{z^2+\rho_{\text{кр}}^2\gamma^2}\end{aligned} \tag{27}$$

The curves of the distribution of current of contact corrosion found are shown in Fig. 4.

CONCLUSIONS

1. Approximate expressions are obtained for the potential and current density of contact corrosion near the boundary of dissimilar metallic surfaces of plane and cylindrical shape under various conditions of coating with paint.

2. The results of the computations are presented in simple engineering formulas and can be used directly in planning systems of anticorrosion protection of various equipment and apparatus.

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