STRUCTURE OF THE ENERGY SPECTRUM OF ELECTRONS IN UNORDERED SYSTEMS

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The structure of the energy spectrum of electrons in nonpolar liquids was investigated in [1]. In [1] we studied two limit cases—"small" and "large" dispersions of the exchange integral. In what follows we shall examine, for the sake of simplicity, a unidimensional model of solid impenetrable balls [2]. The parameter of the problem is $\mu = r_0/2(l - a)$, where r_0 is the distance at which the exchange integral begins to decrease, l is the average distance between particles, and a is the diameter of the ball. The $\mu \ll 1$ case corresponds to the approximation for gases. When $\mu \ll 1$ and we use the method of partial summation of the Newman development of the Green function we obtain for the average state density $\langle\rho\rangle$:

$$\langle\rho\rangle = 2\mu (J_m)^{-2\mu} |E|^{2\mu-1}, \quad |E| \leqslant J_m. \tag{1}$$

In the region $|E| > J_m$ the value of $\langle\rho\rangle$ is zero. Equation (1) can easily be generalized to a tridimensional case. Then one obtains an expression which is in qualitative agreement with the results obtained by Lifshits for concentrational widening of the impurity level [3],

$$\langle\rho\rangle = 2\mu \cdot E^{-1} \cdot \Phi(a) (a + r_0 \ln J_m / E)^2; \quad 0 < E \leqslant J_m, \tag{2}$$

where Φ (R) is the binary distribution function and $\mu = c \cdot \pi r_0$. (In [1] partial summation was made in the mass operator, and this may turn out to be incorrect in some cases.) Equation (1) is not a satisfactory approximation for the vicinity of the point E = 0. In [1] we investigated the case $\mu \gg 1$ by the method of the Green function (excitation theory for the mass operator). However, the method used in [1] is correct only inside the zone. The results obtained in [1] for the vicinity of the edge of the old zone indicate only the tendency of the electron spectrum to expand. In the problem investigated the operator J (see [1]) is the Jacobian matrix. It is well known that there is a close relationship between Jacobian matrices and continuous fractions. In fact, it is this that allowed Dyson [4] to obtain a precise solution of the spectrum of oscillations of a unidimensional chain of atoms with accidental uncorrelated bonds. Our problem is different from Dyson's problem in terms of physical meaning and mathematical relationships. Using the methods of continuous fractions [4], it is possible to obtain an approximate expression for $\langle\rho\rangle$ when $\mu \ll 1$ [which coincides with (1)] as well as when $\mu \gg 1$:

$$\langle\rho\rangle \sim \frac{1}{\pi \sqrt{4J_m^2 - E^2}} \left[1 - \frac{2}{\mu}\left(\frac{E^2}{2J_m^2} - 1\right)\right]; \quad E < 2J_m, \quad \mu \gg 1. \tag{3}$$

LITERATURE CITED

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