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FLUID MECHANICS

ENERGY DISSIPATION IN A TURBULENT GAS CONTAINING SUSPENDED PARTICLES

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A turbulent gas at sufficiently high Reynolds numbers in which solid particles are suspended is considered. An assumption of almost local homogeneity, isotropy, and stationarity is made relative to the turbulent flow. The suspended particles considered here are of dimensions R, sufficiently small compared to the inner scale of turbulence l so that the Reynolds numbers of their motion relative to the gas are less than unity. We considered particle motion under those conditions in a previous paper [1], and the same notation is used here.

The desired quantity ε, the total energy dissipation per unit volume of suspension per unit time, is made up of ε_1, the energy dissipation due to interaction between the gas and the solid particles, and ε_0, the energy dissipation of the pure gas. If n denotes the mean number of particles per unit volume and θ the mean energy dissipation per particle per unit time, then evidently $\varepsilon_1 = n\theta$. In order to evaluate the quantity θ, taking into account that the dissipative portion of the drag of particle motion f_i is given by the Stokes formula, we obtain the relation

$$\theta = \overline{f_i V_i} = k_2 \rho_0 \nu R \overline{V_i(t) V_i(t)} = k_2 \rho_0 \nu R S(0). \qquad (1)$$

It follows from (1) that to find ε_1 it is necessary to know the mean-square velocity of the relative particle motion S(0). A formula permitting the determination of S(0) if $Q(\tau)$, the Lagrangian correlation function of a gas containing suspended particles is known, has been obtained in [1]:

$$S(0) = (1 - \alpha)^2 \left[Q(0) - \beta \int_0^\infty Q(\tau) e^{-\beta\tau} d\tau \right]. \qquad (2)$$

If the assumption is made that the influence of the particles on the gas motion can be neglected (the mixture is considered "passive"), then the correlation function of a pure gas which does not contain suspended particles should be substituted into (2).

In order to clarify the conditions of applicability of the last assumption, let us consider in more detail the nature of particle interaction with a turbulent medium in which they are suspended. Let us recall that the latter consideration is not concerned with the influence of the gravity field, but also assumes that $\gamma \ll 1$, i.e., the mean spacing between particles is much greater than the size of the particles themselves.

The nature of particle interaction with turbulent pulsations of different scales is determined by the ratio of the frequency of pulsation of a given scale size to the characteristic frequency β, dependent on the properties of the particles and on the characteristics of the medium, but independent of the structure of turbulence. If $\beta < \omega_L$, i.e., the particle possesses sufficiently high inertia, even the largest scale pulsations do not succeed in setting it in motion, in practice, during a period. Such a heavy particle is almost immobile, and hence pulsations of all scales flow around it. In the other limiting case, $\beta > \omega_L$, pulsations of all scales entrain the particle, and it tends to follow them all. If the quantity β is such that it is less than ω_l but simultaneously large in comparison with ω_L, it will then be entrained by those pulsations whose frequencies are small compared to β, and it will be followed around by pulsations whose frequencies are large compared to β. Pulsations with frequencies close to β will partially entrain, and particularly flow around the suspended particle.

Evidently, in order that the influence of the presence of particles on the gas motion can be neglected, it is necessary that the term characterizing the force acting from the particle on the gas should be dropped from the equation of gas motion. In other words, in order to be able to consider the impurity "passive" it is necessary that the loss of

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gas momentum due to the particles should be small compared to the total loss of momentum of gas. Mathematically this means that the inequality

$$\frac{\rho_0 \dot{U}}{nf} \sim \frac{\rho_0 \dot{U}}{\rho\gamma\dot{W}} \sim \frac{\alpha}{\gamma}\frac{\dot{U}}{\dot{W}} \gg 1 \tag{3}$$

should be satisfied.

The quantities $\dot{U}$ and $\dot{W}$ in (3) are the accelerations of the gas and the particles, respectively. If we transform to Fourier components in the equation connecting the particle and flow velocities [1], the inequality (3) becomes

$$\frac{\alpha}{\gamma}\sqrt{\frac{\omega^2+\beta^2}{\alpha^2\omega^2+\beta^2}} \gg 1. \tag{4}$$

It follows from condition (4) that if $\alpha/\gamma \gg 1$ (the mass of the suspended particles is small compared to the mass of the flow), then the mixture is always passive since $[(\omega^2+\beta^2)/(\alpha^2\omega^2+\beta^2)]^{1/2} \geqslant 1$ for all ω. If $\alpha/\gamma \sim 1$, inequality (4) is satisfied only for sufficiently high frequencies $\omega \gg \gamma/\alpha \sim \beta$. Therefore, if we consider times much less than $1/\beta$, the mixture may be considered "passive." However, if we are interested in the behavior of the gas over times greater than $1/\beta$ it is impossible to neglect the influence of the particles on the gas motion. This means that the presence in the gas of suspended particles whose mass is comparable to the mass of the gas results in a change in the spectrum of the large-scale pulsations, and does not affect the fine-scale pulsations. In fact, since the large-scale pulsations completely entrain the particles, the particle velocities at large scales are practically equal to the gas velocity at these scales. In other words, the behavior of a gas suspension at scales sufficiently large compared to the amplitudes of the relative particle motion, and over times sufficiently large compared to $1/\beta$ can be considered approximately as a homogeneous medium but with macroscopic parameters different from the parameters of the pure gas. In the opposite limiting case, if we are interested in time intervals small in comparison with $1/\beta$, and scales small in comparison with the amplitudes of the relative particle motion, the particles may be considered as almost immobile, with fine-scale pulsations whose accelerations are much greater than the particle accelerations flowing around them. The loss of momentum by these pulsations to the particles is small compared to their total loss of momentum, and hence the presence of the particles does not affect pulsations whose frequencies are large compared to β. Therefore, if the gas is considered over time intervals which are small compared to $1/\beta$, its characteristics (including the correlation function) will be the same as in a pure gas which does not contain suspended particles.

Upon analyzing the integral in (2) it is easy to note that values of τ small compared to $1/\beta$ yield the main contribution, and hence the correlation function of a pure gas may be used for $Q(\tau)$. In general the form of this function for an arbitrary value of the correlation time is not known. However, since knowledge of the behavior of the function $Q(\tau)$ only for $\tau < 1/\beta$ is required for the determination of $S(0)$, this function may then be determined in the two important limiting cases. If the values of the quantity τ which gives the main contribution to the integral in (2) are small compared with the periods of large-scale pulsations $1/\omega_L$, but nevertheless large compared to the smallest periods of the pulsation spectrum $1/\omega_l$, a known expression [2]

$$Q(\tau) = Q(0) - k_3\frac{\varepsilon_0}{\rho_0}\tau \tag{5}$$

may then be used for the function Q.

Since $\omega_L \sim U_L/L$, $\omega_l \sim U_l/l \sim (\nu\rho_0/\varepsilon_0)^{-1/2}$, then

$$\frac{\omega_L}{\beta} \sim \frac{U_L R^2}{L\alpha\nu} \sim \frac{R^2 U_L L}{\alpha L^2 \nu} \sim \frac{\mathrm{Re}_L R^2}{\alpha l^2 \mathrm{Re}_L^{3/2}} \sim \frac{1}{\alpha \mathrm{Re}_L^{1/2}}\frac{R^2}{l^2},$$

$$\frac{\omega_l}{\beta} \sim \frac{\omega_l}{\omega_L}\frac{\omega_L}{\beta} \sim \mathrm{Re}_L^{1/2}\frac{1}{\alpha \mathrm{Re}_L^{1/2}}\frac{R^2}{l^2} \sim \frac{1}{\alpha}\frac{R^2}{l^2}.$$

It may easily be noticed from the estimates presented that in order to satisfy the requirement $\omega_l^{-1} < \beta^{-1} < \omega_L^{-1}$ needed for using (5), it is necessary that the particle size satisfy the conditions $\alpha^{1/2} < R/l < \alpha^{1/2}\mathrm{Re}_L^{1/4}$. If the particle size is so small that the inequality $R < \alpha^{1/2} l$ is satisfied, then values of τ less than ω_l^{-1} yield the main contribution to the integral in (2). Hence, in place of (5) it is necessary to use another formula [2]

$$Q(\tau) = Q(0) - k_4(\varepsilon_0^3/\rho_0^3\nu)^{1/2}\tau^2. \tag{6}$$

for the correlation function.

Substituting (5) and (6) in (2), and integrating, we obtain values of the mean-square velocities of the relative particle motion:

$$(\overline{V^2})^{1/2} = k_5\frac{\varepsilon_0^{1/2}|1-\alpha|}{\rho_0^{1/2}\alpha^{1/2}\nu^{1/2}}R$$

$$\text{if} \quad \alpha^{1/2} l < R < \alpha^{1/2}\mathrm{Re}_L^{1/4} l; \tag{7}$$

$$(\overline{V^2})^{1/2} = k_6\frac{\varepsilon_0^{3/4}|1-\alpha|}{\rho_0^{3/4}\alpha\nu^{5/4}}R^2 \quad \text{if} \quad R < \alpha^{1/2} l. \tag{8}$$

Comparing (7) and (8), it can easily be seen that they go over into each other for $R \approx \alpha^{1/2} l$. Moreover, taking into account that $\varepsilon_0 / \rho_0 \sim U^3_L / L \sim U^2_L \omega_L$, it is easy to see that (7) is obtained in the limiting case $\omega_L \ll \beta$ from a relationship we derived earlier [1], where an empirical equation was used for the correlation function. The latter equation correctly describes the behavior of the true correlation function for sufficiently large correlation times; however, it becomes inapplicable near the point $\tau = 0$. It is clear from the above why (8) is not obtained in the limit from the corresponding equation in [1].

Using (7), (8), and (1), we find the total energy dissipation per unit volume per unit time:

$$\varepsilon = \varepsilon_0 \left[1 + k_7 (1 - \alpha)^2 \frac{\gamma}{\alpha} \right]$$

$$\text{if} \quad \alpha^{1/2} l < R < \alpha^{1/2} \mathrm{Re}_L^{1/4} l, \qquad (9)$$

$$\varepsilon = \varepsilon_0 \left[1 + k_8 (1 - \alpha)^2 \frac{\gamma}{\alpha} \frac{R^2}{\alpha l^2} \right] \qquad \text{if} \quad R < \alpha^{1/2} l. \qquad (10)$$

From the form of (9) and (10) it follows that if solid particles are suspended in a gas (in this case $\alpha \sim 10^{-3}$), then the additional energy dissipation in the mixture which is due to the particles will become comparable to the dissipation in a pure gas for $\gamma \sim 10^{-3}$. For the case of particles in a fluid $\alpha \sim 1$ and $\varepsilon = \varepsilon_0$, i.e., the presence of a small amount of suspended particles particularly does not alter the magnitude of the dissipated energy.

The results obtained may be interpreted as follows. For a liquid suspension the particle and fluid densities are comparable. Consequently, entrainment of the particles by the fluid will be almost complete and the energy dissipation due to the relative motion of the fluid and particles will be small. If the particles are suspended in a gas, the relative velocity of their motion becomes so great, because of the great difference in the particle and gas densities, that the energy dissipated on the particle surface becomes comparable with the energy dissipated in a volume of pure gas. The fact that (9) does not contain the viscosity of the gas is connected with the fact that viscosity plays a double part in the mechanism of energy dissipation at a particle surface. On the one hand, as the viscosity grows, energy dissipation by each particle increases for a given velocity of relative motion. But, on the other hand, gas entrainment of the particles increases, which results in diminution of the relative velocity, meaning that both effects cancel each other for particles of size $\alpha^{1/2} l < R < \alpha^{1/2} \mathrm{Re}_L^{1/4} l$. Consequently, there will be no explicit dependence of ε on ν in (9). For finer particles the effect of viscosity on the change in relative velocity turns out to be more substantial, and hence, the quantity ε drops as the viscosity grows.

Besides the dissipation of flow energy at the particle surface, which is connected with their inertia, there will be an energy dissipation ε_1' due to particle motion relative to the medium under the influence of the gravity field. Taking into account that the free fall velocity of a particle is $U = g/\beta$, and also using (7) and (8), let us compare the values of ε_1 and ε_1'. If $\alpha^{1/2} l < R < \alpha^{1/2} R_L^{1/4} l$, then

$$\frac{\varepsilon_1}{\varepsilon_1'} = \frac{\overline{V^2}}{U^2} \sim \frac{\beta^2 \varepsilon_0}{\rho_0 g^2 \beta} \sim \frac{\alpha U_l^4}{g^2 R^2} \sim \left(\frac{a_l}{g} \right)^2 \frac{\alpha l^2}{R^2}. \qquad (11)$$

If $R < \alpha^{1/2} l$, then

$$\varepsilon_1 / \varepsilon_1' = \overline{V^2} / U^2 \sim \beta^2 a_l^2 / g^2 \beta^2 \sim (a_l / g)^2. \qquad (12)$$

The a_l in (11) and (12) denotes the acceleration of pulsations of scale l. From the form of (11) and (12) it follows that if the condition $a_l < g$ is satisfied, the inertial energy dissipation of the flow on the particles ε_1 may always be neglected compared with the gravitational dissipation ε_1'. If the reverse inequality $a_l > g$ is satisfied, then inertial dissipation may turn out to be greater than gravitational dissipation.

LITERATURE CITED

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