HYDRODYNAMICS AND MASS TRANSFER IN A TUBE WITH THE LAMINAR FLOW OF VAPOR AND A THIN FILM OF LIQUID

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Film-type rectification and adsorption apparatus with plane-parallel packing is used in modern chemical engineering. It must be noted that a majority of investigations in the field of film-type mass transfer are carried out with turbulent flow of the vapor and pseudolaminar flow of the liquid [1-5].

At the present time, promising compact heat exchangers with laminar flow of both vapor and gas are being developed [6]. Since the laminar flow conditions of a two-phase system are determined not only by the form of the channel, but also by the specific mass flow rates of the phases (which permits optimizing the mass-transfer process), investigations in this field are also desirable. Since, in tubular packing, the flow and mass transfer in which are investigated in the present article, a nonadiabatic heat-transfer process takes place easily, the article, in addition to the above problems, considers also questions of heat transfer from the side of the phase interface and from the side of the wall. As a basis for further investigations, it was considered expedient to examine only a steady-state process in a two-component system, with constant mass flow rates and physical properties of the phases, while the actual dependences of the equilibrium and mean concentrations (C_b and C_m) are assumed to be linear (C_b' and C_m').

Neglecting the molecular transfer of a component and of heat along the channel and the dissipation of energy, the process of the transfer of momentum, mass, and heat in a cylindrical channel may be described by equations of the form

$$\frac{\nu}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right)=\frac{1}{\rho}\frac{dp}{dz}-g,$$
$$v\frac{\partial C'}{\partial z}=\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C'}{\partial r}\right),$$
$$v\frac{\partial T}{\partial z}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

with the boundary conditions

$$dv_g/dr=0,\quad r=0,\quad v_l=0,\quad r=R;$$
$$\nu_g\rho_g\, dv_g/dr=\nu_l\rho_l\, dv_l/dr,\quad v_g=v_l,\quad r=hg;$$
$$\partial C_g'/\partial r=0,\quad r=0,\quad \partial C_l'/\partial r=0,\quad r=R;$$
$$D_g\rho_g\,\partial C_g'/\partial r=D_l\rho_l\,\partial C_l'/\partial r,\quad C_g'/C_l'=c,\quad r=h_g;$$
$$\partial T_g/\partial r=0,\quad r=0,\quad \partial T_l/\partial r=-q_w/a_lC_{pl}\rho_l,\ r=R;$$
$$a_gC_{pg}\rho_g\,\partial T_g/\partial r=a_lC_{pl}\rho_l\,\partial T_l/\partial r,\quad T_g=T_l,\quad r=h_g.$$

In these relationships, a is the thermal diffusivity coefficient, D is the coefficient of mutual diffusion h_g is the radius of the gas flow, p is the pressure, q_w is the specific heat flux from the side of the wall, R is the radius of the tube, (r, z) are cylindrical coordinates, v is the velocity, ν is the kinematic velocity coëfficient, ρ is the density of the mixture, and g and l are the parameters of the gas and liquid phases, respectively.

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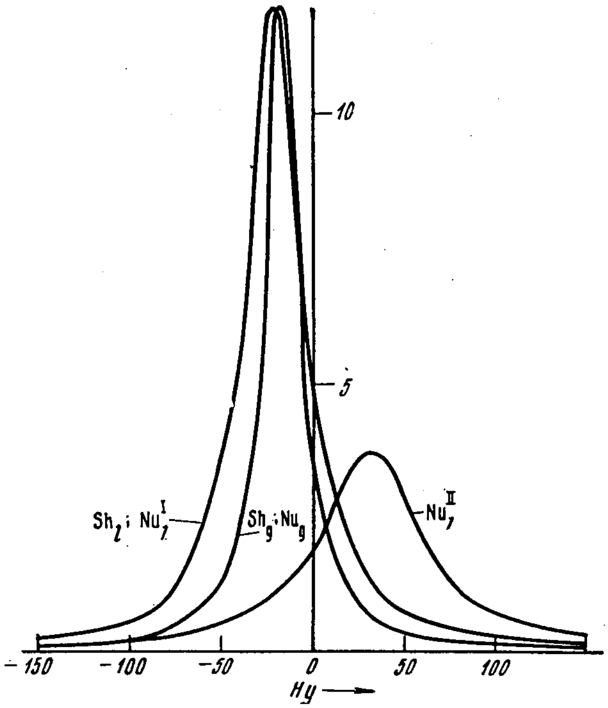

Fig. 1

In addition, for the mean values of C' and of the phase temperature T' with respect to the mass flow rate, calculated from the equilibrium temperature of the flows, the following relationships hold:

$$\frac{C'_{gm}}{C'_{lm}} = -\frac{G_l}{G_g} = -\frac{1}{g},$$

$$\frac{T'_{gm}}{T'_{lm}} = -\frac{C_{pl}G_l}{C_{pg}G_g} = -\frac{1}{c_p g},$$

where

$$G_g = 2\pi\rho_g \int_0^{h_g} v_g r dr = \pi h_g^2 \rho_g \bar{v}_g,$$

$$G_l = 2\pi\rho_l \int_{h_g}^{R} v_l r dr = \pi (R^2 - h_g^2) \rho_l \bar{v}_l.$$

The results of the solution of the starting system are given in the dimensionless coordinates

$$\eta_g = r / h_g, \quad \eta_l = (R - r) / (R - h_g) = (h_g + h_l - r) / h_l,$$
$$\zeta = zD / h^2\bar{v}, \quad \xi = za / h^2\bar{v}$$

and the definitions

$$A = \frac{a_g}{a_l}, \quad d = \frac{D_g}{D_l}, \quad \mathrm{Eu} = -\frac{h^2}{\bar{v}\nu\rho}\frac{dp}{dz}, \quad \mathrm{Er} = \frac{h^2 g}{\bar{v}\nu}, \quad H = \frac{h_g}{h_l},$$

$$Hy = \frac{h^2}{\bar{v}\nu\rho}\left(\rho g - \frac{dp}{dz}\right), \quad Nu_{lw} = \frac{q_w}{a_l C_{pl}\rho_l (T_m - T_w)}, \quad u = \frac{v}{\bar{v}},$$

$$V = \frac{\bar{v}_g}{\bar{v}_l} = \frac{g}{PH}\left(2 + \frac{1}{H}\right), \quad N = \frac{\nu_g}{\nu_l}, \quad P = \frac{\rho_g}{\rho_l}, \quad \varsigma = \frac{C'}{C'_{0m}}, \quad \tau = \frac{T'}{T'_{0m}}.$$

The equation for the continuity of momentum in the gas phase is brought down to the expression

$$u_g = \left(1 + \frac{Hy_g}{8}\right) - 2\frac{Hy_g}{8}\eta_g^2,$$

and, in the liquid film, with an accuracy to a value on the order of 1/H, yields

$$u_l = \left[2\left(1 + \frac{Hy_l}{6}\right)\eta_l - 3\frac{Hy_l}{6}\eta_l^2\right] + \frac{1}{H}\left[-\frac{1}{3}\left(1 + \frac{Hy_l}{6}\right)\eta_l + \left(1 + \frac{Hy_l}{6}\right)\eta_l^2 - \frac{Hy_l}{6}\eta_l^3\right].$$

As a result, on the basis of the boundary conditions, we can find the hydrodynamic criteria determining the form of the velocity profiles of the phases,

$$Hy_g = 2\left[4\frac{g}{PH} - 3\left(1 - \frac{1}{3H}\right)\right] / \left(\frac{g}{PH} + \frac{gN}{H^2}\right),$$

$$Hy_l = 3\left[\left(\frac{g}{PH}\left(1 + \frac{1}{H}\right) + 4\frac{gN}{H^2} - 4\frac{g^2N}{PH^3}\left(1 + \frac{1}{6H}\right)\right)\right] / \left(\frac{g}{PH} + \frac{gN}{H^2}\right).$$

Taking account of the discontinuities of the surfaces and of the mass forces, the Euler and Froude numbers are

$$\mathrm{Eu}_g = \left(Hy_g - \frac{H^2}{NV} Hy_l\right) \Big/ (1-P),\ \mathrm{Eu}_l = \left(Hy_l - \frac{NV}{H^2} Hy_g\right) \Big/ \left(1 - \frac{1}{P}\right),$$

$$\mathrm{Fr}_g = \left(Hy_g - \frac{H^2}{NPV} Hy_l\right) \Big/ \left(1 - \frac{1}{P}\right),\ \mathrm{Er}_l = \left(Hy_l - \frac{NPV}{H^2} Hy_g\right) \Big/ (1-P).$$

The limiting distributions of the concentrations may be found in the form

$$\xi = ke^{\lambda\tau}\sigma(\eta).$$

In this case, the solution of the diffusion equation in the gas can be represented in the form of the series

$$\sigma_g = \sum_{n=0}^{\infty} a_n \eta_g^n,$$

$$a_0 = 1,\quad a_2 = \frac{\lambda}{4}\left(1 + \frac{Hy_g}{8}\right),\ a_{2(n+2)} = -\frac{\lambda_g}{[2(n+2)]^2}\left[2\frac{Hy_g}{8} a_{2n} - \left(1 + \frac{Hy_g}{8}\right) a_{2(n+1)}\right],$$

while the solution of the diffusion in the film without taking account of its curvature yields

$$\sigma_l = \sum_{n=0}^{\infty} b_n \eta^n,$$

$$b_0 = 1,\ b_1 = 0,\ b_2 = 0,$$

$$b_3 = \frac{\lambda}{3}\left(1 + \frac{Hy_l}{6}\right),$$

$$b_{(n+4)} \quad - \frac{\lambda}{(n+4)(n+3)}\left[3\frac{Hy_l}{6} b_n - 2\left(1 + \frac{Hy_l}{6}\right) b_{(n+1)}\right].$$

On the basis of the boundary conditions, we can obtain the following system, determining the eigenvalues:

$$\frac{1}{\sigma_g}\frac{d\sigma_g}{d\eta_g} \Big/ \frac{1}{\sigma_l}\frac{d\sigma_l}{d\eta_l} = -\frac{H}{cdP} \quad \text{at} \quad \eta = 1;\quad \frac{\lambda_g}{\lambda_l} = \frac{H^2V}{d}.$$

In addition, we can write

$$\frac{k_g}{k_l} = -cg\frac{\sigma_l}{\sigma_g} = \frac{gH}{dP}\frac{d\sigma_l}{d\eta_l} \Big/ \frac{d\sigma_g}{d\eta_g} \quad \text{at} \quad \eta = 1.$$

The limiting value of the Sherwood phase criterion

$$\mathrm{Sh} = \frac{1}{(\sigma - \sigma_m)}\frac{d\sigma}{d\eta} \quad \text{at} \quad \eta = 1$$

is a function of Hy and λ:

$$\mathrm{Sh}_g = 16\sum_{n=1}^{\infty} n a_{2n} \Big/ \sum_{n=0}^{\infty}\left[\frac{n(8n + 16 + Hy_g)}{n^2 + 3n + 2}\right] a_n,$$

$$\lim_{H\to\infty} \mathrm{Sh}_l = 6\sum_{n=1}^{\infty} n b_n \Big/ \sum_{n=0}^{\infty}\left[\frac{n(6n + 18 + Hy_l)}{n^2 + 5n + 6}\right] b_n.$$

At $\lambda \to 0$ $\mathrm{Sh}_g = 1536/(Hy_g^2 + 32Hy_g + 384)$ and, up to a value on the order of $1/H$,

$$\mathrm{Sh}_l = \frac{3780}{(Hy_l^2 + 42Hy_l + 756)}\left[1 - \frac{1}{H}\frac{(5Hy_l^3 - 144Hy_l - 504)}{12(Hy_l^2 + 42Hy_l + 756)}\right].$$

Under practical conditions, the correction for the curvature of the film is 1.5%. These dependences, without taking the correction into account, are shown in Fig. 1.

The solution of the energy equation for the gas $\tau_g = k_g e^{\mu_g \xi_g} \sigma_g(\eta_g)$ is analogous to the solution of the diffusion equation ξ_g. It is expedient to seek the solution of the energy equation for the gas in the form

$$\tau_l = \tau_l^{I} + \tau_l^{II} = [k_l^{I}\sigma_l^{I}(\eta_l) + k_l^{II}\sigma_l^{II}(1-\eta_l)]\, e^{\mu_l \xi_l},$$

where the solution τ_l^{I} coincides in form with ξ_l, and τ_l^{II} satisfies the additional condition

$$\partial\tau_l^{II}/\partial(1-\eta_l) = 0 \text{ , at } (1-\eta_l) = 0.$$

Under these circumstances (without correction for the curvature of the film),

$$\sigma_l^{II} = \sum_{n=0}^{\infty} c_n (1-\eta_l)^n,$$

$$c_0 = 1,\ c_1 = 0,\ c_2 = \frac{\mu_l}{2}\left(2 - \frac{Hy_l}{6}\right),\ c_3 = \frac{2\mu_l}{3}\left(\frac{Hy_l}{6} - \frac{1}{2}\right),$$

$$c_{(n+4)} = -\frac{\mu_l}{(n+4)(n+3)}\left[3\frac{Hy_l}{6}c_n - 4\left(\frac{Hy_l}{6} - \frac{1}{2}\right)c_{(n+1)} - \left(2 - \frac{Hy_l}{6}\right)c_{(n+2)}\right].$$

We can introduce the Nu_g and Nu_l^{I} numbers, similar to the Sh_g and Sh_l numbers, respectively and, at $(1-\eta_l)=1$,

$$Nu_l^{II} = \frac{1}{(\sigma_l^{II} - \sigma_{lm}^{II})}\frac{d\sigma_l^{II}}{d(1-\eta_l)} = 3\sum_{n=1}^{\infty} nc_n \Big/ \sum_{n=0}^{\infty}\left[\frac{n(3n^2+18n+24-Hy_l)}{n^3+6n^2+9n+6}\right]c_n;$$

in the limit at $\mu_l \to 0$

$$Nu_l^{II} = 3780/(Hy_l^2 - 63Hy_l + 2016).$$

This dependence is shown in Fig. 1.

It can be shown that in a steady-state process

$$\frac{1}{Nu_{lw}} = \frac{1}{Nu_l^{II}} - \frac{k_l^{I}}{k_l^{II}} \Big/ \left[\frac{d\sigma_l^{II}}{d(1-\eta_l)}\right] \text{ at } (1-\eta_l) = 1,$$

so that, in the given statement of the problem, one of the conditions cannot be given arbitrarily.

On the basis of the boundary conditions, we can find the ratios of the coefficients

$$\frac{k_l^{I}}{k_g} = \frac{AP}{gH}\frac{d\sigma_g}{d\eta_g}\Big/\frac{d\sigma_l^{I}}{d\eta_l} \text{ and } \frac{k_l^{II}}{k_g} = -\frac{AP}{gH}\frac{d\sigma_g}{d\eta_g}\Big/\frac{d\sigma_l^{I}}{d\eta_l} - \frac{1}{c_p g} \quad \text{at} \quad \eta = 1.$$

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