HYDRODYNAMIC MIXING IN A GRANULAR BED. PHYSICAL MODEL OF STAGNANT ZONES

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The cell model of a granular bed, which was proposed by Kramers and Alberda [1], fairly accounts for the observed values of the effective coefficient of longitudinal diffusion in turbulent gas flows. High values for the effective diffusion coefficient and asymmetric distribution functions (with long "tails") of the residence time in the bed are found in liquid flows [2]. Several models of a granular bed with stagnant zones were introduced to explain this phenomenon [3-7]. In a paper by V. G. Levich, V. S. Markin and Yu. A. Chizmadzhev [7] a general analysis of the cell model with stagnant zones was carried out; the actual physical meaning of stagnant zones was not dealt with in this paper, and the volume fraction α of stagnant zones and the specific exchange rate p between the stagnant zone and the main cell volume were introduced as characteristics. Gotschlich [6] suggested that the stagnant zone coincides with the diffusion boundary layer near the surface of solid particles. However, an estimate of the residence time in stagnant zones of this type and the volume of such zones proves that the abnormalities observed in the behavior of liquid flows cannot be explained by this hypothesis. In the present paper it is assumed that the regions near the points where the solid particles contact are the stagnant zones. These regions are equivalent to narrow and deep channels in which turbulent pulsations may be produced. Transfer of substance in the stagnant zones is achieved solely by molecular diffusion, which is the cause of the difference between the behavior of gas and liquid flows. The solution of the problem for a cell system with stagnant zones of the shape mentioned above enables us to explain the shape of the observed distribution functions of the residence time and to obtain estimated values for the effective coefficient of longitudinal diffusion that agree with the experimental data.

The stagnant zone near the point where two spherical particles contact has the shape shown in Fig. 1. The width of the aperture of the stagnant zone is assumed to equal the thickness δ_0 of the viscous layer. Its depth equals $\rho_0 = \sqrt{l\delta_0}$, where l is the particle diameter. Since $l \gg \delta_0$, $\rho_0/\delta_0 = \sqrt{l}/\delta_0 \gg l$. The concentration η of a hydrodynamically neutral admixture in the stagnant zone is found by solving the diffusion equation, which, for a stagnant zone of the shape considered, should be written down in degenerated bipolar coordinates

$$\frac{(\alpha^2+\beta^2)^2}{l^4}\left\{\frac{\partial^2\eta}{\partial\alpha^2}+\frac{\partial^2\eta}{\partial\beta^2}-\frac{\alpha^2-\beta^2}{\alpha(\alpha^2+\beta^2)}\frac{\partial\eta}{\partial\alpha}-\frac{2\beta}{\alpha^2+\beta^2}\frac{\partial\eta}{\partial\beta}\right\}=\frac{1}{D}\frac{\partial\eta}{\partial t}. \quad (1)$$

Here, D denotes the molecular diffusion coefficient; t the time; $\alpha = l^2\rho/(\rho^2+z^2)$, $\beta = l^2 z/(\rho^2+z^2)$ are degenerated bipolar coordinates; ρ, z are cylindrical coordinates. Equation (1) is integrated over the range $0 \leqslant \alpha < \infty$, $-\beta_0 \leqslant \beta \leqslant \beta_0$ with the boundary conditions

$$\eta(\alpha_0, \beta, t) = C(t), \quad \partial\eta/\partial\beta|_{\beta=\pm\beta_0} = 0, \quad (2)$$

where C denotes the concentration in the main (ideally mixed) volume of the cell. Since $\beta \ll \alpha$ in the range considered, it follows from boundary conditions (2) that $\partial\eta/\partial\beta \equiv 0$, and Eq. (1) simplifies to

$$\frac{\partial^2\eta}{\partial\alpha^2}-\frac{1}{\alpha}\frac{\partial\eta}{\partial\alpha}=\frac{l^4}{\alpha^4}\frac{1}{D}\frac{\partial\eta}{\partial t}. \quad (3)$$

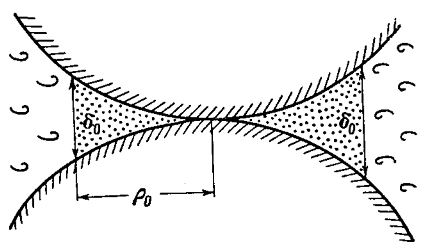

Fig. 1. Shape of the stagnant zone.

We shall write down the equation for the concentration C in the cell and the initial conditions, on the assumption that a δ-pulse of a neutral substance is admixed to the cell at the moment t = 0:

$$\frac{dC}{dt} = -\frac{1}{t}C - D\sigma\,\partial\eta/\partial\rho|_{\rho=\rho_0}, \quad (4)$$

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where $\bar{t}$ denotes the mean residence time in the cell (without stagnant zone), and σ the area of the boundary surface between the cell and the stagnant zone per unit cell volume. The initial conditions for Eq. (3), (4) are

$$C = 1, \quad \eta = 0 \quad \text{at} \quad t = 0. \tag{5}$$

Changing over to the Fourier transforms $\bar{\eta}(\lambda)$, $\bar{C}(\lambda)$, we solve first Eq. (3) and then Eq. (4), taking into account the initial, and boundary conditions (2), (5); this yields:

$$\bar{C}(\lambda) = \left[\bar{t}^{-1} - i\lambda - \frac{2D\sigma}{\rho_0} + \sigma\sqrt{iD\lambda}\, J_0\left(\sqrt{\frac{i\lambda}{D}}\rho_0\right) J_1^{-1}\left(\sqrt{\frac{i\lambda}{D}}\rho_0\right)\right]^{-1}, \tag{6}$$

where J denotes a Bessel function.

But for a normalization factor, the transform $\bar{C}(\lambda)$ coincides with the characteristic distribution function $g_1(\lambda)$ of the residence times in the cell. Introducing the factor $1/\bar{t}$ (which corresponds to normalization of the distribution of unity), we get

$$g_1(\lambda) = \Delta^{-1} = \left[1 - i\lambda\bar{t} - \frac{2D\sigma\bar{t}}{\rho_0} + \bar{t}\sigma\sqrt{iD\lambda}\, J_0\left(\sqrt{\frac{i\lambda}{D}}\rho_0\right) J_1^{-1}\left(\sqrt{\frac{i\lambda}{D}}\rho_0\right)\right]^{-1}. \tag{7}$$

We shall now consider a chain of cells connected in series. The residence times in the various cells are independent. Consequently, to find the characteristic distribution function in a chain of n identical cells, it suffices to raise $g_1(\lambda)$ to the n-th power [8]:

$$g_n(\lambda) = \Delta^{-n}(\lambda). \tag{8}$$

The characteristic function $g_n(\lambda)$ completely characterizes the distribution of the stochastic variable t. The moments of a stochastic variable can be easily found by differentiating the characteristic function. We shall write down the equation for the average m and the central moments μ_k of the distribution function of the residence time in a chain of n cells:

$$m = \frac{1}{i}\frac{dg_n}{d\lambda}\bigg|_{\lambda=0} = n\bar{t}, \tag{9}$$

$$\mu_k = i^{-k}\frac{d^k}{d\lambda^k}\left[g_n(\lambda)e^{-i\lambda m}\right]_{\lambda=0}, \tag{10}$$

$$\mu_2 = n\left[\left(\frac{\Delta'}{\Delta}\right)^2 - \frac{\Delta''}{\Delta}\right]_{\lambda=0} = n\bar{t}^2\left(1 + \frac{1}{48}\frac{\sigma\rho_0^3}{D\bar{t}}\right), \tag{11}$$

$$\mu_3 = n\left[2\left(\frac{\Delta'}{\Delta}\right)^3 - 3\frac{\Delta''\Delta'}{\Delta^2} + \frac{\Delta'''}{\Delta}\right]_{\lambda=0} = n\bar{t}^3\left(2 + \frac{1}{16}\frac{\sigma\rho_0^3}{D\bar{t}} + \frac{1}{256}\frac{\sigma\rho_0^5}{D^2\bar{t}^2}\right). \tag{12}$$

The effective coefficient of longitudinal diffusion D^* is determined by comparing the variance μ_2 with the variance calculated for the model of a quasihomogeneous bed described by the diffusion equation

$$\varepsilon\frac{\partial C}{\partial t} = D^*\frac{\partial^2 C}{\partial z^2} - u\frac{\partial C}{\partial z}, \tag{13}$$

where z is the longitudinal coordinate, and ε the fraction of free volume in the bed. At $uL/D^* \gg 1$ (where $L = nl = n\bar{t}u/\varepsilon$ denotes the bed length), solution of Eq. (13) with the initial condition $C(z, 0) = \delta(z)$ yields a nearly Gaussian distribution with the variance $\mu_2^* = 2LD^*\varepsilon^2/u^3$. Setting μ_2^* and μ_2 equal to each other, we find

$$D^* = \frac{ul}{2}(1 + {}^1/_3 ab), \tag{14}$$

where

$$a = {}^1/_{16}\rho_0^2 / D\bar{t}, \; b = \sigma\rho_0. \tag{15}$$

From (15) it is evident that parameter a is proportional to the ratio between the characteristic residence time ρ_0^2/D in the stagnant zone and the mean residence time $\bar{t}$ in the cell, and that parameter b is proportional to the volume fraction α of the stagnant zones. Formula (14) becomes identical to that derived in paper [7], if we assume that $a/b \sim p/q$.

We shall also write down an expression for the asymmetry coefficient γ of the distribution function considered; this coefficient characterizes the deviation of this distribution from a normal distribution:

$$\gamma = \frac{\mu_3}{\mu_2^{3/2}} = \frac{1}{\sqrt{n}} \frac{2 + ab(1 + a)}{(1 + {}^1/_3 ab)^{3/2}}. \tag{16}$$

We shall estimate the orders of magnitudes of the parameters occurring in formulas (14) and (16). Since $\sigma \sim \delta_0 \rho_0 l^{-3} \approx \delta_0^{3/2} l^{-5/2}$, it follows that

$$a \sim \frac{\delta_0 l}{D} \frac{u}{l} \sim \frac{\nu}{D} \frac{u}{u_*} \sim \Pr k^{-1/2}, \tag{17}$$

$$b \sim \frac{\delta_0^{3/2}}{l^{5/2}} \delta_0^{1/2} l^{1/2} \sim \frac{\delta_0^2}{l^2} \sim \left(\frac{\delta_0 u_*}{\nu} \frac{\nu}{lu}\right)^2 \left(\frac{u}{u_*}\right)^2 \sim \mathrm{Re}^{-2} k^{-1}, \tag{18}$$

where $\mathrm{Re} = lu/\nu$ denotes Reynolds' number; $\Pr = \nu$ D Prandtl's number; $k = (u_*/u)^2$ the dimensionless resistance coefficient; ν the kinematic viscosity, and $u_* = \nu/\delta_0$ the characteristic turbulent velocity. To estimate the order of magnitude of the parameters, we apply the usual theory of turbulence near a wall [9]. Since $\sqrt{k} \sim 10^{-1}$, the product ab in gas flows ($\Pr \sim 1$) is much smaller than unity, and the stagnant zones do not affect the effective diffusion coefficient. In liquid flows ($\Pr \sim 10^3$) at $\mathrm{Re} \sim 10^2$ the product ab equals about unity, which increases D^* several times. At $\mathrm{Re} \sim 10^3$, $ab \ll 1$, even in liquid flows; in this case exactly as in gas flows, $D^* = ul/2$, which coincides with the value of the effective coefficient of longitudinal diffusion in the cell model without stagnant zones [1]. The estimates given are in qualitative agreement with the existing experimental data on the variance of the flow in a granular bed [2, 10]. Since the resistance coefficient k is the essential parameter of the theory, it is desirable to do experiments in which the variation of the effective diffusion coefficient and the pressure drop over the bed are measured simultaneously in order to check the theory more rigorously.

We shall now estimate the asymmetry coefficient γ. In gas flows $ab \ll 1$ and $\gamma \sim 2/\sqrt{n}$, exactly as in a chain of cells without stagnant zones. Under these conditions, $\gamma \ll 1$, even in a moderate number of cells, and the distribution established is nearly normal. Since $a \gg 1$, three situations are possible in liquid flows. At $ab \gtrsim 1$ ($\mathrm{Re} \sim 10^2$)

$$\gamma \sim \frac{1}{\sqrt{n}} \sqrt{\frac{a}{b}} \sim \frac{1}{\sqrt{n}} \Pr^{1/2} \mathrm{Re}\, k^{1/4} \gg \frac{1}{\sqrt{n}}. \tag{19}$$

A normal distribution would establish only if $n \sim \Pr \mathrm{Re}^2 k^{1/2} \sim 10^6$, i.e., nonsymmetric distribution functions with long "tails" will practically always be observed. If $ab \ll 1$, then,

$$\gamma \sim \frac{1}{\sqrt{n}} a^2 b \sim \frac{1}{\sqrt{n}} \Pr^2 \mathrm{Re}^{-2} k^{-2}. \tag{20}$$

At $\mathrm{Re} \sim 10^3$, when the stagnant zones no longer contribute to the variance, the asymmetry may be even more considerable, since at a moderate number of cells the "tails" of the distribution function can disappear only if $\mathrm{Re} \sim 10^4$-10^5. Consequently, it cannot be considered correct to describe the variance of a liquid flow in a granular bed at moderate values of Reynolds number by a diffusion equation of type (13).

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